MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXVIII. JANUARY 13, 1928. No. 3

Rev. T. E. R. PHILLIPS, PRESIDENT, in the Chair.

Herbert Brown, The Haven, 11 Walsingham Road, Enfield, Middlesex;

Lt.-Col. George Dudley MacMahon, The Orchard, Church Crookham, Hants;

Mrs. Louise Victoria Mathieson, 6 Park Circus Place, Glasgow; and

Frederick Gordon Wood, M.A., Headmaster, Caldicott School, Hitchin, Herts,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

F. C. R. Douglas, M.A., 8 Cambridge Road, Battersea, London, S.W. 1 (proposed by L. Becker);

John Leslie Haughton, D.Sc., F.Inst.P., National Physical Laboratory, and 7 Bolton Gardens, Teddington (proposed by J. Guild);

Frederick Neville Hopkins, 31 Raglan Road, Retford, Notts (proposed by W. J. Coombs);

David Percival Larham, Lieut. R.N.R., 49 Portswood Road, Southampton, Hants (proposed by R. E. Beare);

C. P. Sivarama Menon, B.A., 112 Gower Street, London, W.C. 1 (proposed by L. N. G. Filon);

Dr. Rolf Müller, Astrophysical Observatory, Potsdam, Germany, and Casilla 649, La Paz, Bolivia (proposed by P. Guthnick);

Philip Augustus Taylor, Ph.D., Mathematical Department, Imperial College of Science and Technology, and 8 Wyndham Place, Bryanston Square, London, W. 1 (proposed by S. Chapman);
Dr. Heinrich Vogt, Professor of Astronomy, Königstuhl Observatory, Heidelberg, Germany (proposed by Max Wolf);
Evan Gwyn Williams, B.A., 71 Eardley Crescent, London, S.W. 5 (proposed by C. C. L. Gregory); and
C. J. Wilson, Mount Faber Observatory, Singapore (proposed by P. J. Melotte).

Eighty-two presents were announced as having been received since the last meeting, including amongst others :—

Astrographic Catalogue: Tacubaya Observatory, Tomo I, zona $-15°$; Specola Vaticana, vol. ix., $+56°$. Appendix I. and II.
Astrographic Chart, Paris Observatory, zones $+18°$, $+20°$ (4 charts).
W. Bowie, Isostasy.
Meudon Observatory, Annales, Tome 7.
Harvard College Observatory, Two glass positives of the Greater and Lesser Magellanic Clouds.

The President announced that the Gold Medal of the Society had been awarded to Professor R. A. Sampson, for his Theory of the Four Great Satellites of Jupiter; and that a Jackson-Gwilt Medal and Gift had been awarded to Mr. W. Reid, for his discovery of six new comets; and to Dr. W. H. Steavenson, for his work on faint variable stars, novæ, and the Herschel instruments.

The President further announced that Dr. W. H. Wright, of Lick Observatory, had been appointed George Darwin Lecturer for 1928.

Discussion of some further Observations of β Lyræ.

By Mary A. Blagg.

This paper is supplementary to two which have previously appeared in *M.N.* (**84**, 629, and **85**, 484), dealing with observations of β Lyræ by Baxendell (1840–77) and by four members of the B.A.A. (1906–20).

Colonel Markwick, shortly before his death, very kindly entrusted to me the original observations of the same star made by B.A.A. members from 1898 to 1905, *i.e.* from the beginning of its being under observation by them. I also gratefully acknowledge the receipt of similar observations made by E. F. Sawyer (1877 October to 1882 November—omitting 1880); and by the Rev. J. G. Hagen, S.J. (1887 April to August); obtained for me by Professor H. H. Turner's kind-

ness.* All these have proved of great value in testing and carrying forward the conclusions arrived at in regard to a secondary cycle of variation in the light of β Lyræ, which I have called the "B" wave. They go a considerable way towards filling in the gap between 1877 and 1906. To save printing, I do not repeat here explanations of the methods employed which have already been given, nor go so fully into details as before.

The Baxendell examples have been re-worked, using for the epoch of principal minimum my revised formula:

$$2398590{\cdot}57 + 12{\cdot}908006E + 0{\cdot}00000391 4E^2;$$

and also rearranging the B-wave examples by making the assumed mean cycle 6·57 days in agreement with that used in the B.A.A. series, the starting-points of the examples being altered in accordance. This necessarily changes the actual figures, but theoretically the *date* of the minimum phase should remain unchanged. It is satisfactory that (omitting the last Baxendell example, which alone is sensibly affected by the revised formula) the average alteration is less than $\pm$ 0^d·2, and none exceeds 0^d·4. No date, however, is absolutely unchanged, and these small differences indicate errors due to the irregularity and scantiness of the observations. Two examples, the old 21 and 22, have been discarded, as being quite untrustworthy, but two new ones (now Nos. 5 and 8) have been obtained. Some few examples have been revised by reducing or increasing the number of cycles included in them. Certain small differences from the figures in the previously given B.A.A. tables are thus accounted for.

The same mean curve for β Lyræ's principal variation was used for the new B.A.A. observations as for the former ones, but the Sawyer mean curve differed slightly from it in a regular, symmetrical manner. Baxendell's curve was used for the Hagen observations. The differences are not great, however, between any of these curves.

The same formula (that given above) was used with all the observations, and was found to fit them all. It is the *residuals* from the mean curve which are tabulated to obtain the data for the B wave.

The B.A.A. observations of 1898–1905 were made by sixteen observers, many of them beginners at the work. Eleven of them provided too small a number of observations to supply trustworthy evidence of personal peculiarities. These, with two exceptions, were therefore not used. One exception consists of C. L. Brook, a practised observer, whose regular observations proved easy to deal with. The other (the late W. E. Besley) is included because his observations are the only ones available for 1898, and they form the first B.A.A. example. But as he gave merely his deduced magnitude (to 0·1 m.) without any details as to how it was obtained, this example cannot be regarded as having any real importance. Of these seven observers, Colonel Markwick (Ma.) and Professor Moye (Mo.) were included in the previously discussed series. Similar formulæ to those applied to them for correction of personality were used for C. L. Brook (Br.), J. W. L. Child (Ch.), and J. M. Péridier (Pr.), but none was needed for the Rev. J. T. Bird (Bd.).

* Professor Turner has also given me much valuable advice.

TABLE I.

(The unit for the six residuals is 0·01 magnitude.)

New No.	Old No.	No. of Obs.	Observers.	(1).	(2).	(3).	(4).	(5).	(6).	Apparent Date of Minimum.
										239+
1	1	13	Baxendell	0	0	+ 1	+ 2	+ 7	+11	3746·50
2	2	19	,,	+ 5	+ 3	+ 5	+10	+ 7	+ 5	6320·91
3	3	28	,,	−13	− 2	+ 5	+ 6	− 4	−15	6549·77
4	4	30	,,	− 6	+ 6	+11	+ 9	0	−10	6602·11
5	..	22	,,	− 6	− 5	0	+ 1	0	− 4	6951·25
6	5	28	,,	−12	− 3	− 1	+ 6	+ 5	− 4	7003·90
7	6	22	,,	+ 5	−12	+ 3	+10	+13	+ 8	7372·73
8	..	14	,,	− 6	− 8	− 3	− 2	+ 8	− 3	7707·60
9	7	17	,,	−11	− 1	0	+ 2	+ 3	+ 1	8028·95
10	8	15	,,	− 4	− 8	+ 2	+ 3	+ 6	+ 4	8476·25
11	9	25	,,	−14	− 9	− 6	− 1	− 6	− 6	8758·09
12	10	19	,,	+ 5	− 1	− 3	+ 6	+ 9	+11	8864·73
13	11	18	,,	−16	−15	+ 1	+ 6	+10	− 2	9126·34
14	12	24	,,	+ 5	+ 3	+ 2	+ 3	+10	+13	9193·42
15	13	28	,,	− 7	− 3	− 4	− 4	+ 4	− 4	9271·15
16	14	20	,,	+ 4	+ 1	− 6	−11	− 3	− 2	9437·90
17	15	20	,,	+11	+ 5	+ 1	+ 3	+ 9	+14	9555·25
18	16	30	,,	+ 1	−10	−15	− 8	− 9	− 5	9831·46
19	17	13	,,	0	+ 5	− 9	−10	+ 1	+ 9	9989·23
										240+
20	18	18	,,	+ 5	+ 7	+ 5	0	− 8	− 2	0201·21
21	19	18	,,	− 4	+ 7	+ 6	− 4	− 5	−13	0556·57
22	20	17	,,	− 7	− 4	+ 3	+ 1	− 5	−12	0912·08
23	23	14	,,	− 9	− 6	+ 1	+ 2	+ 1	− 6	6878·43
24	..	22	Sawyer	− 6	− 7	− 4	+ 2	+ 5	− 3	6951·43
25	..	20	,,	− 2	− 2	− 3	− 1	+11	+ 2	7214·81
26	..	17	,,	+ 5	+ 1	− 1	− 4	− 8	− 2	7283·03
27	..	24	,,	− 3	− 7	− 5	− 8	− 7	− 7	7532·58
28	..	30	,,	− 1	+10	0	+ 3	− 3	0	8269·33
29	..	16	,,	0	+ 1	+ 6	+ 2	− 2	− 6	8316·11
30	..	28	,,	− 2	− 1	0	+ 3	− 2	− 3	8088·98
31	..	30	,,	− 5	0	+ 3	+ 2	− 1	− 4	8625·59
32	..	22	,,	− 2	+ 1	+ 2	+ 1	− 5	− 6	8677·29
										241+
33	..	12	Hagen	+ 1	+11	+18	+ 1	− 3	− 6	0421·90
34	..	13	Besley	− 4	− 7	− 2	+11	+ 6	− 9	4506·23
35	..	14	Ma.	−14	−11	−10	− 5	− 2	− 6	4855·21

TABLE I.—*continued.*

New No.	Old No.	No. of Obs.	Observers.	(1).	(2).	(3).	(4).	(5).	(6).	Apparent Date of Minimum.
										241+
36	..	16	Ma. and Ch.	+ 7	+ 7	− 5	−10	− 3	+ 2	5160·02
37	..	34	Ch. and Pr.	− 3	0	− 1	− 4	− 7	− 4	5286·04
38	..	33	Ma. and Ch.	+ 3	− 2	+ 8	+ 3	− 3	− 8	5602·24
39	..	26	Pr.	− 9	− 2	− 1	0	− 3	− 3	5635·96
40	..	17	Ch. and Pr.	− 5	+ 5	+ 6	+ 8	+ 4	+ 1	5878·98
41	..	23	Ma. and Ch.	− 3	+ 2	− 1	+12	+ 6	+ 7	6077·14
42	..	12	Ma. and Pr.	+ 6	+ 4	+10	+16	+17	+ 7	6306·85
43	..	11	Ma.	+ 2	+ 1	+ 6	+ 6	+13	+ 3	6418·69
44	..	33	Ma. and Br.	− 2	0	+ 2	+ 8	+ 2	+ 4	6700·92
45	..	48	Ma., Mo., and Br.	0	− 5	− 9	− 8	− 3	+ 3	6801·48
46	..	27	Ma. and Bd.	− 2	− 3	0	+ 5	+14	+ 6	7010·41
47	..	25	Ma., Mo., and Br.	+ 5	− 3	− 5	− 4	− 2	+ 5	7123·70
48	..	29	Ma., Mo., and Bd.	+ 7	+ 2	− 9	− 6	+ 1	+ 7	7163·19
49	1	21	Ma., Mi., and Ry.	+ 6	− 4	− 4	− 3	− 1	+ 9	7373·18
50	2	41	,,	+ 3	− 1	− 7	− 5	+ 3	+ 7	7451·93
51	3	49	Ditto. and Mo.	+ 5	− 3	− 4	− 6	− 6	+ 3	7531·46
52	4	17	Ry.	+14	+ 5	− 3	− 8	− 2	+ 8	7682·68
53	5	46	Ma., Mo., and Ry.	+ 1	+ 8	+ 8	+ 9	+ 4	+ 2	7757·54
54	6	22	Ma. and Ry.	− 7	− 1	+ 6	+ 7	+ 1	− 4	7863·12
55	7	21	,,	− 5	− 5	+ 2	+ 9	+ 9	− 1	8093·83
56	8	54	Ditto. and Mo.	− 3	− 5	− 9	− 3	− 2	− 1	8239·87
57	9	27	Ry.	+ 7	0	− 3	0	+ 3	+10	8444·02
58	10	19	Ma.	+ 9	+ 3	− 4	− 5	− 3	+ 8	8523·51
59	11	32	Ma., Mo., and Ry.	+11	+ 7	− 3	−15	−10	+ 6	8629·05
60	12	35	Ma. and Mo.	+15	+22	+16	+15	+12	+ 7	8880·14
61	13	17	Mo.	− 6	− 3	0	− 6	− 9	− 6	8972·15
62	14	21	,,	−12	− 6	+ 2	+ 7	+ 6	+ 3	9223·91
63	15	19	,,	−16	−18	+ 4	+14	+13	+ 3	9585·33
64	16	24	Ma. and Mo.	− 2	− 7	+ 6	+ 6	+ 7	+ 2	9703·56
65	17	39	,,	− 6	−10	− 7	− 1	+10	− 5	9953·71
										242+
66	18	25	Ma.	− 2	− 8	− 7	− 6	− 1	+ 3	0053·28
67	19	23	Mo.	+ 1	−17	−20	+ 3	+16	+14	0335·12
68	20	25	,,	+ 3	−13	−12	+ 3	+ 6	+10	0703·38
69	21	20	,,	−14	− 7	+ 9	+ 2	−10	−17	1167·41
70	22	41	,,	− 5	− 9	− 5	+ 5	+ 7	+ 1	1491·05
71	23	48	Ma. and Mo.	+ 6	− 2	− 6	− 3	+ 5	+ 6	1814·32
72	24	21	Ma.	+ 4	+ 2	− 2	− 6	−23	−12	2223·76
73	25	18	,,	−17	−13	− 2	− 2	+ 3	−10	2561·26

TABLE II.

New No.	E.	Calculated Date for Minimum.	O–C$_1$ (Days).	CF$_1$ (Amp. ·8) (Days).	O–C$_2$ (Days).	CF$_2$ (Amp. ·8) (Days).	O–C$_3$ (Days).
	−	239+					
1	2036	3743·86	+2·64	+ ·76	+1·88	+ ·32	+1·56
2	1644	6320·35	+ ·56	+ ·05	+ ·51	− ·78	+1·29
3	1609	6550·44	− ·67	+ ·35	−1·02	− ·80	− ·22
4	1601	6603·03	− ·92	+ ·42	−1·34	− ·80	− ·54
5	1548	6951·47	− ·22	+ ·73	− ·95	− ·78	− ·17
6	1540	7004·06	− ·16	+ ·76	− ·92	− ·77	− ·15
7	1484	7372·24	+ ·49	+ ·76	− ·27	− ·69	+ ·42
8	1433	7707·57	+ ·03	+ ·50	− ·47	− ·58	+ ·11
9	1384	8029·76	− ·81	+ ·10	− ·91	− ·44	− ·47
10	1316	8476·91	− ·66	− ·48	− ·18	− ·22	+ ·04
11	1273	8759·67	−1·58	− ·73	− ·85	− ·06	− ·79
12	1257	8864·88	− ·15	− ·78	+ ·63	− ·01	+ ·64
13	1217	9127·94	−1·60	− ·78	− ·82	+ ·14	− ·96
14	1207	9193·71	− ·29	− ·76	+ ·47	+ ·17	+ ·30
15	1195	9272·63	−1·48	− ·72	− ·76	+ ·22	− ·98
16	1170	9437·04	+ ·86	− ·59	+1·45	+ ·30	+1·15
17	1152	9555·43	− ·18	− ·47	+ ·29	+ ·36	− ·07
18	1110	9831·60	− ·20	− ·12	− ·08	+ ·49	− ·57
19	1086	9989·51	− ·28	+ ·10	− ·38	+ ·56	− ·94
		240+					
20	1054	0200·00	+1·21	+ ·38	+ ·83	+ ·63	+ ·20
21	1000	0555·19	+1·38	+ ·71	+ ·67	+ ·73	− ·06
22	946	0910·40	+1·68	+ ·79	+ ·89	+ ·79	+ ·10
Average residuals, Nos. 1 to 22			± ·82		± ·75		± ·53
				(Amp. 1·4)		(Amp. ·2)	
23	39	6879·34	− ·91	− ·70	− ·21	− ·14	− ·07
24	28	6951·77	− ·34	− ·54	+ ·20	− ·13	+ ·33
	+						
25	12	7215·13	− ·32	+ ·08	− ·40	− ·10	− ·30
26	22	7280·97	+2·06	+ ·24	+1·82	− ·09	+1·91
27	60	7531·17	+1·41	+ ·80	+ ·61	− ·06	+ ·67
28	172	8268·66	+ ·67	+1·34	− ·67	+ ·04	− ·71
29	179	8314·75	+1·36	+1·30	+ ·06	+ ·05	+ ·01
30	190	8387·19	+1·79	+1·22	+ ·57	+ ·06	+ ·51
31	226	8624·26	+1·33	+ ·85	+ ·48	+ ·09	+ ·39
32	234	8676·94	+ ·35	+ ·74	− ·39	+ ·09	− ·48
		241+					
33	499	0422·30	− ·40	− ·88	+ ·48	+ ·20	+ ·28
Average residuals, Nos. 23 to 33			± ·99		± ·54		± ·51

TABLE II.—*continued.*

New No.	E.	Calculated Date for Minimum.	O–C$_1$ (Days).	CF$_1$ (Amp. 1·4) (Days).	O–C$_2$ (Days).	CF$_2$ (Amp. ·2) (Days).	O–C$_3$ (Days).
	+	+241					
34	1119	4507·47	−1·24	+ ·13	−1·37	− ·20	−1·17
35	1172	4856·79	−1·58	+ ·90	−2·48	− ·20	−2·28
36	1218	5159·99	+ ·03	+1·31	−1·28	− ·19	−1·09
37	1237	5285·23	+ ·81	+1·39	− ·58	− ·19	− ·39
38	1285	5601·64	+ ·60	+1·28	− ·68	− ·17	− ·51
39	1290	5624·60	+1·36	+1·25	+ ·11	− ·17	+ ·28
40	1327	5878·51	+ ·47	+ ·88	− ·41	− ·14	− ·29
41	1357	6076·28	+ ·86	+ ·47	+ ·39	− ·12	+ ·51
42	1392	6307·01	− ·16	− ·08	− ·08	− ·10	+ ·02
43	1409	6419·09	− ·40	− ·35	− ·05	− ·08	+ ·03
44	1452	6702·58	−1·66	− ·94	− ·72	− ·05	− ·67
45	1467	6801·47	+ ·01	−1·11	+1·12	− ·03	+1·15
46	1499	7012·45	−2·04	−1·34	− ·70	− ·01	− ·69
47	1516	7124·53	− ·83	−1·39	+ ·56	+ ·01	+ ·55
48	1522	7164·09	− ·90	−1·40	+ ·50	+ ·02	+ ·48
49	1554	7375·09	−1·91	−1·32	− ·59	+ ·04	− ·63
50	1566	7454·21	−2·28	−1·25	−1·03	+ ·05	−1·08
51	1578	7533·34	−1·88	−1·15	− ·73	+ ·07	− ·80
52	1601	7684·99	−2·31	− ·91	−1·40	+ ·08	−1·48
53	1612	7757·53	+ ·01	− ·77	+ ·78	+ ·09	+ ·69
54	1628	7863·03	+ ·09	− ·54	+ ·63	+ ·11	+ ·52
55	1663	8093·83	·00	·00	·00	+ ·13	− ·13
56	1685	8238·91	+ ·96	+ ·35	+ ·61	+ ·15	+ ·46
57	1716	8443·34	+ ·68	+ ·79	− ·11	+ ·16	− ·27
58	1728	8522·46	+1·05	+ ·94	+ ·11	+ ·17	− ·06
59	1744	8627·97	+1·08	+1·12	− ·04	+ ·18	− ·22
60	1782	8878·58	+1·56	+1·37	+ ·19	+ ·19	·00
61	1796	8970·91	+1·24	+1·40	− ·16	+ ·19	− ·35
62	1834	9221·53	+2·38	+1·30	+1·08	+ ·20	+ ·88
63	1889	9584·29	+1·04	+ ·75	+ ·29	+ ·20	+ ·09
64	1907	9703·01	+ ·55	+ ·50	+ ·05	+ ·19	− ·14
65	1945	9953·64	+ ·07	− ·10	+ ·17	+ ·18	− ·01
		242+					
66	1960	0052·58	+ ·70	− ·33	+1·03	+ ·18	+ ·85
67	2006	0356·01	− ·89	− ·97	+ ·08	+ ·15	− ·07
68	2059	0705·63	−2·25	−1·37	− ·88	+ ·12	−1·00
69	2129	1167·42	− ·01	−1·16	+1·15	+ ·06	+1·09
70	2178	1490·68	+ ·37	− ·57	+ ·94	+ ·02	+ ·92
71	2227	1813·96	+ ·36	+ ·19	+ ·17	− ·03	+ ·20
72	2289	2223·03	+ ·73	+1·05	− ·32	− ·08	− ·24
73	2340	2559·55	+1·71	+1·38	+ ·33	− ·12	+ ·45
Average residuals, Nos. 34 to 73			± ·98		± ·60		± ·54
„ „		whole series	± ·93		± ·63		± ·55

In the present tables actual Julian dates (old form) are given for the minima, and the residuals are expressed in days instead of degrees, which makes the meaning clearer, and lessens the likelihood of slips or misapprehensions. Two such occurred in the former work. (1) "N" was treated as if synonymous with the usual "E," which was not precisely the case. The effect of this was not important. (2) A slip was made in the last B.A.A. example. Neither could have occurred under the present system.

[*Note.*—Three misprints escaped correction. In the first paper (p. 656) in Table VII., No. 7, position of minimum should be 357°; in the second paper (p. 490), Table IV., No. 10, No. of observations should be 19; and (p. 492) Table V., No. 8, under (6), for − 5 read − 3.]

From the new B.A.A. observations, treated exactly like the former ones, fifteen examples of the B wave were obtained—Nos. 34 to 48. They fitted in with the former ones, both in regard to their average period (nearly 6·595 days) and to the apparent fluctuation, shown in column $O - C_1$. The increase in length of period from the time of Baxendell, when it was about 6·576 days, was thus confirmed, and attempts were made to fix the rate of the increase. It proved possible, however, only to place it between about ·0000055 and ·0000067 days per cycle. In these circumstances the Sawyer and Hagen observations were of great value (Nos. 24 to 33), as they come almost exactly half way in date between those of Baxendell and of the B.A.A. Baxendell's last (hitherto isolated) example comes in closely before the first Sawyer one.

Mr. Sawyer writes that when his observations were taken he was inexperienced, and found the star a difficult one. He marks certain observations as "poor." It was thought better not to omit these, but it is not surprising that some of the nine examples obtained are irregular in form. Some of his 277 observations were insufficiently dated or too scattered for use. The Hagen observations were fourteen in number, with two gaps of four weeks each. All except the two latest in date (which occur after the second long gap) were used in forming the one example. Against all probability this example turns out to be quite a good one. These examples show an average period of about 6·585 days—just about half way between the two other values, in agreement with the theory of an increasing period.

They did not, however, suffice to fix definitely the mean rate of increase, though they narrowed the limits of uncertainty. For the present the expression $+ \cdot0000308E^2$ has been adopted (as giving the best results) for the correction to the date of minimum, implying an increase of something over ·000006 days per cycle.

In Table I. the first column gives the new No., the second the old No., the next two the No. of observations and the observers by whom they were taken. Then follow the mean figures obtained in the six "portions" of each example, each portion including 60° of the cycle. The final column gives the apparent date of the minimum. These dates are, of course, not actually *observed*—such observation would be an absolute impossibility—but they are obtained from the figures in the six sections by harmonic analysis in the usual manner, the degrees being then converted into days. The said figures are residuals from the mean curve of principal variation, the unit being 0·01 m., as already stated.

In Table II., E represents the actual number of minima of the B wave assumed to have occurred from the adopted zero date, 2407136·12. (Owing to the fairly rapid increase in the length of the period, a difference of 100 in E in the earlier part represents approximately *two days less* than the same difference in the latter part.) The next column gives the dates found by the formula :

$$2407136{\cdot}12 + 6{\cdot}584E + {\cdot}00000308E^2;$$

6·584 being the approximate mean period for the whole series, and the zero date that which seems best to correspond with it. Both this date and the coefficient of E^2 were obtained partly by trial and error, and either or both may need correction when more evidence is available.

$O - C_1$ gives the difference between the final column of Table I. and this last. Then follows the calculated value of an assumed fluctuation in 552 cycles, for each example. The $O - C_1$ column indicates clearly the existence of some such fluctuation, but its *precise* length is uncertain. By analysis, almost equally good results are obtained with anything from 550 to 554 cycles. In the Baxendell paper a fluctuation of 552 cycles was assumed, and in the second paper one of 504; those "cycles" being fixed ones of 6·454 and 6·57 days respectively. The new observations show that the second is decidedly too short, and the one now adopted is longer than the first one, the *mean* cycle being 6·584 days. Its mean length is therefore 3634·368 days, or nearly ten years, and it increases together with the increase in the period. Its (positive) maximum is taken at $E = 145 \pm 552$.

[*Note.*—Since these results were obtained, my attention has been called to an article in the *Ast. Nach.* of May last, by S. Tscherny, on β Lyræ, in which he announces a theoretically found fluctuation, in the epoch of principal minimum, of 3439·543 days. He calls attention to this being within three days of the arithmetical mean of the values given in my two former papers for the chief fluctuation in the epoch of minimum in the secondary or B wave. He also states that this theoretical fluctuation must increase in length with the increase of the variation period. The "swing" of his fluctuation is very nearly the same as that of mine. Although these fluctuations do not represent exactly the same thing, and although mine (as now adopted) is longer than his, it seems highly probable that my observational fluctuation is connected with his theoretical one.]

The amplitude (or swing) of this fluctuation *appears* to be greater in the middle and later examples than in the Baxendell ones. Although this effect is very possibly due to some outside cause, different values have been given to the amplitude in Table II., viz. 0·8 days for Nos. 1 to 22, and 1·4 days for the rest. $O - C_2$ gives the difference between $O - C_1$ and CF_1. At least one further fluctuation seems to be indicated, and one of 1380 cycles ($2\frac{1}{2}$ times the other) has been adopted experimentally in the column CF_2, the residuals from it being given in the final column. The amplitude of this also differs in the two divisions, being taken as 0·8 days and 0·2 days; the sum of the two amplitudes in each division being 1·6 days. Its (positive) maximum is taken at $E = 470 \pm 1380$. It is suggested that possibly each of these fluctua-

tions may be, not a simple sine curve, but a complex wave compounded of two or more simple ones of different lengths. Various other fluctuations have been tried, some with fairly good results, notably fluctuations of 690 and of 440 or 450 cycles. But while these much improve some parts of the series, they do not, on the whole, make any great difference. Attempts at actually fixing the true fluctuations must be left until further evidence is available, both from observations and from theory. In the residuals, approximately $0^{d}.55 = 30°$, $1^{d}.1 = 60°$, and $1^{d}.6455 = 90°$.

TABLE III.

(a) *Examples with Max. and Min. nearly at one of the Points.*

New No.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	Year and Season.	Sum of Portions.
1	0	− 1	0	+ 1	+ 1	+ 1	+ 2	+ 5	+ 7	+12	+11	+ 5	1841 W	+ 44
2	+ 5	+ 5	+ 5	+ 3	+ 3	+ 4	+ 5	+ 8	+10	+11	+ 7	+ 6	1848 W	+ 72
3	− 4	−10	−15	−16	−13	− 8	− 2	+ 3	+ 5	+ 7	+ 6	+ 2	1849 S	− 45
5	− 2	− 4	− 5	− 6	− 6	− 5	− 2	0	+ 1	+ 1	+ 1	0	1850 S	− 27
7	+ 6	+ 5	− 6	−12	− 8	+ 3	+ 8	+10	+12	+13	+12	+ 8	1851 W	+ 51
12	+ 5	+ 2	− 1	− 3	− 3	+ 2	+ 6	+ 8	+ 9	+11	+11	+ 8	1855 W	+ 55
13	− 2	−10	−16	−17	−15	−10	+ 1	+ 4	+ 6	+10	+10	+ 5	1856 S	− 34
16	− 2	− 6	−10	−11	− 8	− 3	− 3	− 2	+ 2	+ 4	+ 3	+ 1	1857 S	− 35
18	−10	−13	−15	−14	− 8	− 8	− 9	− 7	− 5	0	+ 1	− 2	1858 S	− 90
20	+ 3	0	− 6	− 8	− 6	− 2	+ 2	+ 5	+ 6	+ 7	+ 6	+ 5	1859 S	+ 12
Mean	0	− 3	− 7	− 8	− 6	− 3	+ 1	+ 3	+ 5	+ 8	+ 7	+ 4	(S−W=−8)	$\left(+\frac{3}{12}\right)$
23	− 3	− 6	− 8	− 9	− 8	− 6	− 2	+ 1	+ 2	+ 2	+ 2	+ 1	1877 W	− 34
27	− 6	− 5	− 7	− 8	− 8	− 7	− 7	− 7	− 5	− 3	− 5	− 7	1879 S	− 75
28	+ 2	+ 3	0	− 3	− 2	0	0	− 1	+ 5	+10	+ 5	0	1881 S	+ 19
29	− 1	− 2	− 4	− 6	− 3	0	0	+ 1	+ 4	+ 6	+ 5	+ 2	1881 W	+ 2
31	− 1	− 3	− 4	− 6	− 5	− 3	0	+ 2	+ 3	+ 3	+ 2	+ 1	1882 S	− 11
34	− 3	− 9	− 5	− 4	− 6	− 7	− 5	− 2	+ 6	+11	+10	+ 6	1898 SW	(− 8)
36	+ 3	− 5	− 9	−10	− 8	− 3	0	+ 2	+ 5	+ 7	+ 8	+ 7	1900 S	− 3
38	0	− 3	− 6	− 8	− 3	+ 3	0	− 2	+ 4	+ 8	+ 6	+ 3	1901 S	+ 2
42	+ 7	+ 7	+ 6	+ 5	+ 4	+ 7	+10	+14	+16	+17	+17	+13	1903 S	+123
46	+ 2	− 2	− 3	− 3	− 2	0	+ 2	+ 5	+11	+14	+12	+ 6	1905 S	+ 42
47	− 3	− 4	− 5	− 5	− 4	− 3	− 2	+ 2	+ 5	+ 6	+ 5	+ 1	1905 W	− 7
48	+ 2	− 5	− 9	− 9	− 6	− 3	+ 1	+ 5	+ 7	+ 8	+ 7	+ 5	1905 W	+ 3
Mean, subtracting $4\frac{1}{2}$ from each total.	0	− 3	− 5	− 6	− 5	− 2	− 1	+ 1	+ 5	+ 7	+ 6	+ 3	(S−W=+2)	$\left(+\frac{53}{12}\right)$

TABLE III.—*continued.*

New No.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	Year and Season.	Sum of Portions.
52	+ 1	− 3	− 7	− 8	− 6	− 2	+ 4	+ 8	+12	+14	+10	+ 5	1907 S	+ 28
55	− 1	− 4	− 5	− 6	− 5	− 3	+ 2	+ 7	+ 9	+10	+ 9	+ 4	1908 S	+ 17
60	+15	+13	+12	+ 8	+ 7	+10	+15	+19	+22	+21	+16	+15	1910 S	+173
61	− 6	− 8	− 9	− 8	− 6	− 6	− 6	− 5	− 3	− 1	0	− 2	1910 W	− 60
62	+ 3	− 6	−12	−10	− 6	− 1	+ 2	+ 5	+ 7	+ 7	+ 6	+ 5	1911 S	0
63	+ 3	− 7	−16	−18	−18	−12	+ 4	+11	+14	+15	+13	+10	1912 S	− 1
64	+ 2	0	− 2	− 5	− 7	− 1	+ 6	+ 6	+ 6	+ 7	+ 7	+ 5	1912 W	+ 24
65	− 6	− 6	− 9	−10	−10	− 7	− 4	− 1	+ 7	+10	+ 5	− 5	1913 S	− 36
67	+ 1	− 8	−17	−20	−20	−12	+ 3	+12	+16	+16	+14	+ 7	1914 S	− 8
68	+ 3	− 8	−13	−14	−12	− 4	+ 3	+ 5	+ 6	+ 9	+10	+ 7	1915 S	− 8
71	+ 3	− 2	− 5	− 6	− 5	− 3	+ 2	+ 5	+ 6	+ 6	+ 6	+ 6	1918 SW	(+ 13)
72	− 3	− 6	−16	−23	−19	−12	− 2	+ 4	+ 3	+ 2	0	− 2	1919 W	− 74
Mean, subtracting 5⅔ from each total.	+ 1	− 4	− 9	−10	− 9	− 5	+ 2	+ 6	+ 8	+ 9	+ 8	+ 4	(S–W=+5)	$\left(+\frac{68}{12}\right)$
General mean, subtracting 10 from each total.	0	− 4	− 7	− 8	− 7	− 3	+ 1	+ 4	+ 6	+ 8	+ 7	+ 4	..	$\left(+\frac{124}{12}\right)$

(b) *Examples with Max. and Min. nearly Half-way between Points.*

4	0	− 7	−10	−10	− 6	+ 1	+ 6	+10	+11	+11	+ 9	+ 5	1849 S	+ 20
6	− 4	− 8	−12	− 8	− 3	− 2	− 1	+ 3	+ 6	+ 6	+ 5	+ 1	1850 W	− 17
8	− 5	− 6	− 8	− 8	− 7	− 3	− 2	− 2	+ 6	+ 8	+ 4	− 3	1852 S	− 26
9	+ 1	− 5	−11	− 8	− 1	0	0	+ 1	+ 2	+ 3	+ 3	+ 2	1853 S	− 13
10	0	− 4	− 8	− 8	− 4	+ 2	+ 2	+ 3	+ 5	+ 6	+ 5	+ 4	1854 W	+ 3
11	− 6	− 8	−14	−13	− 9	− 7	− 6	− 3	− 1	− 2	− 6	− 6	1855 S	− 81
14	+ 4	+ 3	+ 2	+ 2	+ 3	+ 3	+ 7	+10	+13	+13	+ 9	+ 5	1856 SW	(+ 74)
15	− 6	− 7	− 6	− 3	− 3	− 4	− 4	− 4	+ 3	+ 4	+ 1	− 4	1856 W	− 33
17	+ 5	+ 2	+ 1	+ 1	+ 3	+ 6	+ 9	+12	+14	+13	+11	+ 8	1857 W	+ 85
19	+ 5	− 4	− 9	−11	−10	− 5	+ 1	+ 7	+ 9	+ 6	0	+ 4	1858 W	− 7
21	− 4	− 5	−10	−13	−10	− 4	+ 3	+ 7	+ 7	+ 6	0	− 4	1860 S	− 27
22	− 5	− 9	−12	−12	− 7	− 6	− 4	0	+ 3	+ 3	+ 1	− 2	1861 S	− 50
Mean, adding 6 to each total.	− 1	− 4	− 8	− 7	− 4	− 1	+ 1	+ 4	+ 7	+ 7	+ 4	+ 1	(S–W=−3)	$\left(-\frac{72}{12}\right)$

TABLE III.—*continued.*

New No.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	Year and Season.	Sum of Portions.
24	− 5	− 6	− 7	− 7	− 6	− 4	− 1	+ 2	+ 5	+ 5	+ 1	− 3	1877 W	− 26
25	− 2	− 2	− 2	− 3	− 3	− 2	− 1	+ 5	+11	+ 9	+ 2	− 1	1878 SW	(+ 11)
26	− 2	− 4	− 7	− 8	− 6	− 2	0	+ 5	+ 4	+ 1	0	− 1	1878 W	− 20
30	− 2	− 3	− 3	− 2	− 2	− 1	− 1	0	+ 2	+ 3	+ 1	− 2	1881 W	− 10
32	− 2	− 5	− 7	− 6	− 4	− 2	0	+ 1	+ 2	+ 2	+ 1	+ 1	1882 W	− 19
33	− 2	− 3	− 5	− 6	− 3	+ 1	+ 5	+11	+17	+18	+10	+ 1	1887 S	+ 44
35	− 9	−14	−13	−11	−11	−10	− 8	− 5	− 2	− 2	− 3	− 6	1899 SW	(− 94)
37	− 4	− 6	− 7	− 6	− 4	− 3	− 3	− 2	0	0	− 1	− 2	1900 W	− 38
39	− 3	− 3	− 7	− 9	− 7	− 2	− 2	− 1	0	0	− 1	− 3	1901 W	− 38
40	+ 3	+ 1	− 3	− 5	− 1	+ 5	+ 6	+ 6	+ 7	+ 8	+ 7	+ 4	1902 S	+ 38
41	+ 2	− 3	− 1	+ 2	+ 1	− 1	+ 5	+12	+ 9	+ 6	+ 6	+ 7	1902 W	+ 45
43	+ 2	+ 2	+ 1	+ 1	+ 3	+ 6	+ 6	+ 6	+12	+13	+10	+ 3	1903 W	+ 65
44	+ 4	+ 1	− 2	− 1	0	+ 1	+ 2	+ 5	+ 8	+ 7	+ 2	+ 3	1904 SW	(+ 30)
45	− 5	− 7	− 9	− 9	− 8	− 6	− 3	0	+ 3	+ 3	0	− 3	1904 W	− 44
Mean, adding $4\frac{3}{4}$ to each total.	− 1	− 3	− 5	− 5	− 3	− 1	+ 1	+ 4	+ 6	+ 6	+ 3	0	(S–W=+4)	$\left(-\frac{56}{12}\right)$
49	− 4	− 4	− 4	− 3	− 3	− 2	− 1	+ 5	+ 9	+ 8	+ 6	− 2	1906 S	+ 5
50	− 1	− 5	− 7	− 7	− 5	− 1	+ 3	+ 6	+ 7	+ 6	+ 3	+ 1	1906 W	0
51	− 4	− 4	− 5	− 6	− 6	− 6	− 3	+ 3	+ 5	+ 5	+ 2	− 3	1906 W	− 22
53	+ 4	+ 3	+ 2	+ 1	+ 1	+ 4	+ 8	+ 8	+ 8	+ 9	+ 9	+ 7	1907 S	+ 64
54	− 2	− 4	− 6	− 7	− 5	− 1	+ 3	+ 6	+ 7	+ 7	+ 4	+ 1	1907 W	+ 3
56	− 4	− 5	− 8	− 9	− 7	− 3	− 2	− 2	− 1	− 1	− 2	− 3	1908 W	− 47
57	0	− 2	− 3	− 2	0	+ 2	+ 3	+ 7	+10	+10	+ 7	+ 4	1909 S	+ 36
58	− 1	− 4	− 5	− 5	− 4	− 3	+ 2	+ 8	+ 9	+ 9	+ 7	+ 3	1909 S	+ 16
59	− 3	−12	−15	−15	−10	− 1	+ 6	+10	+11	+10	+ 7	+ 2	1909 W	− 10
66	− 7	− 8	− 8	− 7	− 6	− 6	− 4	− 1	+ 2	+ 3	+ 2	− 2	1913 W	− 42
69	−10	−14	−17	−18	−14	−11	− 7	+ 5	+ 9	+ 8	+ 2	− 4	1916 W	− 71
70	− 2	− 5	− 8	− 9	− 8	− 5	+ 1	+ 5	+ 8	+ 7	+ 4	+ 1	1917 W	− 11
73	−10	−15	−17	−15	−13	− 9	− 2	− 2	− 2	+ 3	+ 3	− 2	1920 W	− 81
Mean, adding $13\frac{1}{4}$ to each total.	− 2	− 5	− 7	− 7	− 5	− 2	+ 2	+ 5	+ 7	+ 7	+ 5	+ 1	S–W=+5)	$\left(-\frac{160}{12}\right)$
General mean, adding 24 to each total.	− 2	− 4	− 6	− 6	− 4	− 1	+ 1	+ 4	+ 7	+ 7	+ 4	+ 1	..	$\left(-\frac{288}{12}\right)$

As before, the individual examples were plotted and intermediate points read off between each of the six given in Table I. This was done entirely afresh for the old examples (necessarily in the case of Baxendell), and slight differences in Nos. 49 to 72 are thus explained. No. 73 was changed in position.

The examples were, as before, divided into sets (*a*) and (*b*), according to the position of the phases, and were tabulated in such a way as to bring the maximum and minimum of each as nearly as possible together, without altering any of the figures, the object being to gain some idea of the form and amplitude of the wave.

In Table III. the twelve portions of each example are followed by the year and season, and by the sum of the portions. "S" includes April to August 15; "W," August 16 to January. Both (*a*) and (*b*) are subdivided into three sections, chronologically. At the foot of each is given the mean for each portion, and the mean value of "S – W." This value is, approximately, for Nos. 1 to 22 (1841–61), – 5; for Nos. 24 to 32 (1877–82), $-\frac{1}{2}$; for Nos. 36 to 73 (1900–20), + 4. Baxendell's last example is omitted because it belongs in *date* to the second division, and Hagen's because it is his only one. These figures do not support the primarily "seasonal" character of the differences, which may rather be mainly due to a subsidiary variation completed in *nearly* a year. This question deserves investigation, which must, however, await further evidence. It is nevertheless clear that altitude, orientation, and brightness of the sky have a certain effect for some personalities.

A few of the examples in my former papers are, in this table, changed from (*a*) to (*b*), or *vice versa*, on fresh consideration or revision. The general means: (*a*) 0, – 4, – 7, – 8, – 7, – 3, + 1, + 4, + 6, + 8, + 7, + 4; and (*b*) – 2, – 4, – 6, – 6, – 4, – 1, + 1, + 4, + 7, + 7, + 4, + 1, are but little altered by the additional matter and revision. There is again a near approach to a sine curve. The average amplitude in the Baxendell examples is 7 (·07 m.); in the Sawyer and Hagen ones, 5 (·05 m.); in the 1898–1905 B.A.A. ones, 6 (·06 m.); and in the 1906–20 ones, over 8; the general average being 7. Owing to many different causes, the amplitude (the wave being real) would tend to come out too small, and it may probably be actually about a tenth of a magnitude. Such irregular examples as Nos. 15, 18, 19, 27, 28, 38, 41, 44, and 61 have a bad effect upon the averages. No. 27 is the only one which may be said to give no real evidence of the wave.

The Hagen example is given on page 174 as an illustration.

These observations would have been regarded as too much scattered for use, had they not been the only ones available between 1882 and 1898.

[Table

	(1).	(2).	(3).	(4).	(5).	(6).
a	..	..	..	..	..	−7
b	..	+11	..	..	..	..
c	+2	..	..	..	..	..
d	..	..	..	..	..	..
e	..	..	+25	..	..	..
f	+1	..	..	+8	..	..
g	..	..	..	..	..	..
h	..	..	..	..	..	..
j	..	..	..	..	..	..
k	..	..	..	..	−3	−4
l	..	..	+12	..	..	..
m	+2	..	..	..	..	..
n	..	..	..	..	..	..
o	−2	..	..	−6	..	..
Mean	+1	+11	+18	+1	−3	−6

Summary.

1. Some hundreds of observations of β Lyræ taken by seven members of the B.A.A. in the years 1898–1905, and other observations of the same star by E. F. Sawyer, 1877–82, and by the Rev. J. G. Hagen, S.J., in 1887, have been dealt with in a manner similar to that adopted in my two previous papers on this star, using the same formula for the principal variation as in the second paper.

2. The subsidiary wave of light variation previously discovered (called B) was again evident in these further observations, and its apparent mean period in the Sawyer and Hagen observations was 6·585 days, half-way between those found in the earlier and later series, thus confirming the theory of a gradual increase in the period of this wave, similar to that in the principal wave of variation. The advance in the epoch of minimum is provisionally assumed to be + ·00000308E², but its *precise* value is still uncertain.

3. The chief fluctuation in the phases previously noted was again apparent, and is now estimated at 552 cycles, the swing each way *appearing* to be greater in the Sawyer and B.A.A. series than in that of Baxendell. A second fluctuation, in 1380 cycles, seems also to be indicated, with a wider swing in the Baxendell series than in the later ones. It is suggested that such apparent inconsistencies may be due to the effect of other fluctuations. Meanwhile the values assumed for these are only tentative.

4. Examples of this subsidiary (B) wave have been given, tracing it (though with many gaps) through 4376 of its cycles, and they must, I think, be held to prove its real existence and its gradually increasing length, though many details may require revision.

Spectroscopic Parallaxes of B-type Stars (Fifth Paper): The Parallaxes of 125 Stars. By D. L. Edwards, A.R.C.S., D.I.C.

(*Communicated by the Director of the Norman Lockyer Observatory.*)

The parallaxes given in the accompanying tables are in continuation of similar lists previously published.* The two methods which are used for their determination have already been described in these papers; it is sufficient here to remark that the first method depends upon photometric measures of the intensities of certain Helium and Hydrogen lines by means of a graduated optical wedge, the second being based on careful classification both of spectral type and line character. The first of these methods is applicable only to a limited section of B-type stars, in which the spectral lines are strong enough and sufficiently well defined to be measurable by the wedge. It is not applicable to stars with wide hazy lines, bright lines, or other similar peculiarities, and only in exceptional cases to stars of spectral type later than B5. The second method can be used for practically all B-type stars, with only a few possible exceptions, some of which are given in Table IV. In wedge measurements the personal equation appears to be slowly variable; and it has been found necessary to measure, in addition to the stars given in this paper, a number of standard stars of well-determined parallaxes. These standard stars were used in checking (and, where necessary, in redetermining) the reduction curves required for converting measures into absolute magnitudes.

The columns in the following tables are practically self-explanatory, except that M_1 and M_2 are the spectroscopic absolute magnitudes by the first and second method respectively; π_1 and π_2 are the corresponding spectroscopic parallaxes.

In Table I. are given the results of wedge measurements of stars not hitherto treated by this method, but for which parallaxes by the second method have previously been published. The spectroscopic absolute magnitudes and parallaxes as determined by the wedge are given in columns 7 and 8. The next two columns give other parallaxes (in units of 0″·001) for the sake of comparison, column 9 being the previously published parallaxes by the second method. Authorities for the parallaxes in column 10 are given in the final column, in which W = Mount Wilson, g = group parallax.

Table II. gives the classifications, spectroscopic absolute magnitudes, and parallaxes of stars by the second method (columns 7 to 10) of stars previously measured by the wedge but not classified by the second method. Column 11 gives the previously published wedge parallaxes for comparison (in units of 0″·001). Table III. gives classifications, absolute magnitudes, and parallaxes—by both methods when possible—of stars not previously dealt with by either method.

The notes to Tables II. and III. consist, firstly, of other classifications as given by Adams and Joy,† Lindblad and Schalén,‡ and

* *Monthly Notices*, **84**, 366; **85**, 439; **87**, 364.
† *Ap. J.*, **57**, 294. ‡ *Upsala Meddelanden*, No. 17.

Abetti ; * secondly, of parallaxes from these and other sources for comparison.

Special difficulties arise in the attempt at the classification of certain stars, owing to marked peculiarities in their spectra. Such stars do not all show the same type of peculiarity, and have to be considered individually, taking into account any possible explanation of their variations from the normal in attempting to assign a reasonable classification. A list of such stars is given in Table IV. These stars are placed in what appear to be the most probable classifications, and the corresponding absolute magnitudes and parallaxes are added in columns 9 and 10. It must be remarked that these classifications and the resulting parallaxes are only approximate, depending in some cases on hypothetical reasons for the peculiarities. They represent the best that can be done with these stars by this method. The notes to Table IV. include (in addition to other classifications and parallaxes) remarks on the peculiarities of the individual stars of the table. It may be noticed that two of these stars (HD32549 and HD111133) show a peculiarity common to a group of peculiar spectra mentioned by Dr. H. C. Woods † in his work on spectroscopic parallaxes of A-type stars. This peculiarity is the extreme faintness of the [K] line in relation to the strength of other metallic lines in the spectra. On the other hand, a third star (HD212593, 4 Lac.) has a [K] line rather stronger than might be expected for its spectral type (B8)—possibly on account of the sharpness of this line.

Table I.

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spec. (H.D.).	7. Spectroscopic. M_1.	8. Spectroscopic. π_1.	9. π_2.	10. Other Values. π.	11. Other Values. Auth.
144	10 Cas.	0 1·2	+63 37	5·5	B8	0·0	0·008	8	10	W.
3379	53 Psc.	31·6	+14 41	5·9	B3	−1·7	·003	3	3	W.
4636	ν Cas.	43·2	+50 25	5·0	B9	+0·1	·010	9	9	g.
14372	− And.	2 14·2	+46 51	6·1	B5	−1·2	·004	3	4	W.
17081	π Cet.	39·4	−14 17	4·4	B5	−0·5	·011	11		
28149	72 Tau.	4 21·3	+22 46	5·4	B5	−0·8	·006	6	7	W.
34816	λ Lep.	5 15·0	−13 17	4·3	B1	−2·2	·005	4	6	g.
35671	115 Tau.	21·3	+17 53	5·3	B3	−1·3	·005	6	8	g.
35708	114 Tau.	21·6	+21 51	4·8	B3	−2·4	·004	4		
36371	χ Aur.	26·2	+32 7	4·9	B1	−2·1	·004	4		
36819	121 Tau.	29·3	+23 58	5·3	B3	−1·7	·004	5		
37018	42 Ori.	30·4	− 4 54	4·7	B3	−1·9	·005	5	6	W.
37756	− Ori.	35·8	− 1 11	5·0	B3	−1·5	·005	6	7, 6	W, g.
38622	133 Tau.	42·0	+13 52	5·2	B5	−1·6	·004	5		
39291	55 Ori.	46·5	− 7 33	5·3	B3	−2·0	·004	5		

* *Pub. R. Univ. Firenze,* fasc. 42.
† *Monthly Notices,* **87**, 389.

TABLE I —*continued.*

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. m.	6. Spec. (H.D.).	7. Spectroscopic. M_1.	8. Spectroscopic. π_1.	9. π_2.	10. Other Values. π.	11. Other Values. Auth.
		h m	° ′							
40967	3 Mon.	57·1	−10 36	5·0	B8	−1·1	·006	4		
48977	16 Mon.	6 41·1	+ 8 42	5·8	B3	−1·7	·003	4		
49340	43 Cam.	42·9	+69 0	5·1	B5	−0·3	·008	7		
50707	15 CMa.	49·2	−20 6	4·7	B1	−2·5	·004	4		
53974	− CMa.	7 2·0	−11 8	5·3	B3	−2·1	·003	3		
67880	− Pup.	8 4·9	−15 57	5·5	B3	−1·9	·003	4	4	W.
81188	$\varkappa$ Vel.	9 19·0	−54 35	2·6	B3	−1·5	·015	16	11	g.
83754	$\varkappa$ Hya.	35·5	−13 53	5·0	B3	−0·4	·008	8	8	W.
87015	− Leo.	57·2	+22 26	5·6	B3	−1·5	·004	5		
89688	23 Sex.	10 15·9	+ 2 47	6·5	B3	−1·5	·003	3	3	W.
105382	− Cen.	12 2·9	−50 6	4·8	B5	−0·9	·007	7	9	g.
127972	η Cen.	14 29·2	−41 43	2·7	B3p	−1·4	·015	14	19, 13	W, g.
136298	δ Lup.	15 14·8	−40 17	3·4	B2	−1·5	·010	10		
136504	ε Lup.	15 9	−44 20	3·7	B3	−1·3	·010	9	7	g.
138764	− Lib.	29·0	− 8 51	5·2	B5	−0·7	·007	6	6	W.
142669	ϱ Sco.	50·7	−28 55	4·0	B3	−1·8	·007	10	8	g.
143275	δ Sco.	54·4	−22 20	2·5	Bo	−2·0	·013	10	11	g.
176871	− Lyr.	18 57·2	+26 9	5·5	B3	−0·7	·006	6		
178125	18 Aql.	19 2·3	+10 55	5·1	B8	−0·6	·007	9		
179761	21 Aql.	8·7	+ 2 7	5·1	B8	−0·5	·008	7		
180968	2 Vul.	13·5	+22 51	5·4	Bo	−1·9	·003	3	6	W.
184606	9 Vul.	30·2	+19 33	4·9	B8	−0·2	·010	11		
184915	$\varkappa$ Aql.	31·5	− 7 15	5·0	Bo	−2·2	·004	4	7	W.
188665	23 Cyg.	51·2	+57 16	5·0	B5	−1·2	·006	6		
205139	− Cep.	21 28·3	+60 1	5·5	Bo	−2·8	·002	2		
220222	64 Peg.	23 17·0	+31 16	5·4	B8	−0·8	·006	5		
224151	− Cas.	50·5	+56 53	6·1	Bo	−3·0	·002	2	2	W.

TABLE II.

1. H.D.	2. Name.	3. 1900. R.A.	4. 1900. Dec.	5. m.	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M_2.	10. π_2.	11. π_1.
		h m	° ′							
1606	− And.	0 15·2	+30 22	5·8	B8	B5	*ns*	−0·7	·005	5
10144	α Eri.	1 34·0	−57 45	0·6	B5	B3	*ns*	−1·4	·040	41
16908	35 Ari.	2 37·6	+27 17	4·6	B3	B3	*ns*	−1·4	·006	5
21071	− Per.	3 18·9	+48 45	5·9	B5	B7	*ns*	−0·2	·006	5
21291	− Cam.	21·0	+59 36	4·4	B9p	B8	*ss*	−1·3	·007	12
23466	29 Tau.	40·3	+ 5 44	5·4	B3	B4	*ns*	−1·0	·005	5

TABLE II.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
H.D.	Name.	1900. R.A.	Dec.	*m.*	Spectrum. H.D.	N.L.O.		M_2.	π_2.	π_1.
		h m	° ′							
23793	30 Tau.	42·8	+10 50	5·0	B3	B3	*s*	−1·8	·004	3
25204	λ Tau.	55·1	+12 12	var.	B3	B3	*s*	−1·8	·009	7
29335	49 Eri.	4 32·1	+ 0 48	5·3	B5	B6	*s*	−0·8	·006	7
32630	η Aur.	59·5	+41 6	3·3	B3	B4	*s*	−1·3	·012	10
34503	τ Ori.	5 12·8	− 6 57	3·7	B5	B6	*s*	−0·8	·013	11
35497	β Tau.	20·0	+28 31	1·8	B8	B7	*ss*	−1·4	·023	34
37490	ω Ori.	33·9	+ 4 4	4·5	B3*p*	B3	*ns*	−1·4	·007	4
42545	69 Ori.	6 6·2	+16 9	4·9	B3	B3	*n*	−1·1	·006	6
52918	19 Mon.	57·9	− 4 6	4·9	B3	B3	*n*	−1·1	·006	6
147394	τ Her.	16 16·7	+46 33	3·9	B5	B4	*s*	−1·3	·009	13
182568	2 Cyg.	19 20·2	+29 26	4·9	B3	B3	*s*	−1·8	·005	4
198183	λ Cyg.	20 43·5	+36 7	4·5	B5	B5	*ns*	−0·7	·009	12
200595	− Cyg.	59·3	+45 27	6·2	B8	B4	*ns*	−1·0	·004	4
209952	α Gru.	22 1·9	−47 27	2·2	B5	B5	*ns*	−0·7	·026	40
211924	30 Peg.	15·5	+ 5 17	5·4	B5	B5	*s*	−1·0	·005	6
217050	− Lac.	52·7	+48 9	5·2	B3	B2	*n*	−1·6	·004	6
217101	− Lac.	53·1	+38 48	6·1	B3	B3	*s*	−1·8	·003	3
217675	*o* And.	57·3	+41 47	3·6	B5	B5	*ns*	−0·7	·014	13
218440	− Cas.	23 3·0	+59 13	6·3	B3	B3	*s*	−1·8	·002	3
223128	− Cep.	41·8	+66 13	5·9	B3	B3	*s*	−1·8	·003	3
223229	− And.	42·6	+46 16	5·8	B3	B3	*ns*	−1·4	·004	3

Notes on Table II.

Order of Remarks: First, other classifications (if any); second, other parallaxes. Authorities given in brackets.

Abbreviations: W = Mt. Wilson, U = Lindblad and Schalén (Upsala), Ab = Abetti, g = group parallaxes, t = trigonometric, d = dynamical. (*N.B.*—Parallaxes previously published by the Wedge method are given in column 11, above.)

10144: π = ·049 (t).
21291: B–Aoτ (U); ·003 (U).
25204: π = − ·008 (t).
29335: B7*n* (W); ·007 (W).
32630: B3*s* (W), B–Aoσ + (U); ·020 (g), ·014 (t), ·011 (W), ·016 (U).
34503: B7*s* (W); ·004 (t), ·013 (W).
35497: B9*s* (W), Bσ + (U); ·024 (t), ·040 (W), ·032 (U).
37490: π = ·005 (g).
198183: π = ·027 (t).
200595: π = ·005 (t).
209952: π = ·022 (t).
211924: B6*s* (W); ·005 (W).
217050: Bτ − (U); ·004 (U).
217675: B7*n* (W), Bτ − (U); ·003 (t), ·019 (W), ·008 (U)

TABLE III.

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M_1.	10. π_1.	11. M_2.	12. π_2.
2011	12 Cas.	0 19·3	+61 17	5·4	B9	B9	*ns*	..	..	0·0	·008
12301	53 Cas.	1 55·6	+63 54	5·6	B5*p*	B6	*s*	−1·4	·004	−0·8	·005
18883	– Cet.	2 57·1	+ 3 58	5·6	B5	B7	*s*	−0·5	·006	−0·6	·006
21790	17 Eri.	3 25·6	− 5 25	4·8	B9	B9	*ns*	..	..	0·0	·010
24504	– Per.	48·8	+47 35	5·3	B5	B4	*n*	−0·7	·006	−0·7	·006
24640	– Per.	50·0	+34 47	5·5	B3	B3	*s*	−1·7	·004	−1·8	·004
25558	40 Tau.	58·4	+ 5 9	5·3	B3	B4	*ns*	−1·3	·005	−1·0	·005
28375	44 Eri.	4 23·4	+ 1 9	5·5	B8	B6	*ns*	..	..	−0·5	·006
50658	ψ^9 Aur.	6 49·1	+46 25	5·8	B8	B7	*ns*	..	..	−0·2	·006
58661	– Lyn.	7 21·5	+48 24	5·6	B9*p*	B8	*ns*	..	..	−0·1	·007
60325	– Pup.	28·8	−14 7	6·2	B5	B3	*ns*	−1·1	·004	−1·4	·003
60855	– Pup.	31·4	−14 16	5·6	B5	B3	*n*	−0·9	·005	−1·1	·005
71906	– Lyn.	8 24·8	+37 36	6·1	B9	B9	*ns*	..	..	0·0	·006
74393	– Hya.	38·7	+ 4 42	6·3	B8	B9	*ns*	..	..	0·0	·005
87901	α Leo	10 3·0	+12 27	1·3	B8	B8	*n*	..	..	+0·2	·060
89239	– Leo	12·6	+27 55	6·5	B9	B9	*ns*	..	..	0·0	·005
89774	42 Leo	16·5	+15 29	6·1	B9	B9	*s*	..	..	−0·3	·005
92728	39 UMa.	37·4	+57 43	5·8	B9	A0	*s*	..	..	−0·1	·007
106625	γ Crv.	12 10·7	−16 59	2·8	B8	B7	*s*	..	..	−0·6	·021
139892	ζ CrB.	15 35·6	+36 58	5·1	B8	B6	*n*	..	..	−0·3	·008
142926	4 Her.	52·2	+42 51	5·6	B8	B8	*n*	..	..	+0·2	·008
144470	ω' Sco.	16 1·0	−20 24	4·1	B2	B3	*ns*	−1·7	·007	−1·4	·008
145389	φ Her.	5·6	+45 12	4·3	B9*p*	A0	*s*	..	..	−0·1	·013
145502	ν Sco.	6·2	−19 12	4·3	B3	B3	*ns*	−1·6	·007	−1·4	·007
146926	19 UMi.	13·7	+76 8	5·5	B8	B9	*ns*	..	..	0·0	·008
149121	28 Her.	27·6	+ 5 44	5·6	B8	B9	*s*	..	..	−0·3	·007
164900	97 Her.	17 58·3	+22 56	6·1	B8	B4	*n*	..	..	−0·7	·004
170650	– Her.	18 25·5	+23 48	5·7	B5	B5	*ns*	−1·1	·004	−0·7	·005
175863	..	52·3	+59 53	6·9	B5	B5	*ns*	−0·9	·003	−0·7	·003
179406	20 Aql.	19 7·3	− 8 7	5·4	B3	B3	*n*	−1·3	·005	−1·1	·005
185507	σ Aql.	34·3	+ 5 10	5·2	B3	B2	*s*	−1·9	·004	−2·3	·003
185936	– Aql.	19 36·5	+13 35	5·8	B3	B4	*s*	−1·1	·004	−1·3	·004
187879	– Cyg.	47·2	+40 20	5·6	B2	B3	*ns*	−1·8	·003	−1·4	·004
188252	– Cyg.	49·2	+47 41	5·7	B2	B3	*ss*	−2·0	·003	−2·3	·003
189944	– Vul.	57·5	+24 31	5·8	B8	B6	*s*	−1·0	·004	−0·8	·005
190603	– Cyg.	20 0·7	+31 56	5·7	B0	B1	*ss*	−3·0	·002	−3·2	·002

TABLE III —*continued.*

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M_1.	10. π_1.	11. M_2.	12. π_2.
194636	π Cap.	21·6	−18 32	5·2	B8	B7	*s*	..	..	−0·6	·007
196775	− Del.	34·4	+15 29	5·9	B2	B3	*ns*	−1·7	·003	−1·4	·004
197392	− Cyg.	38·3	+41 22	5·6	B8	B7	*s*	..	..	−0·6	·006
198667	5 Aqr.	46·8	− 5 52	5·5	B8	B8	*n*	..	..	+0·2	·009
199478	− Cyg.	52·4	+47 2	5·8	B8*p*	B7	*ss*	..	..	−1·4	·004
202753	15 Aqr.	21 12·9	− 4 57	5·7	B8	B5	*s*	..	..	−1·0	·005
207971	γ Gru.	47·9	−37 50	3·2	B8	B8	*s*	..	..	−0·5	·018
210424	38 Aqr.	22 5·2	−12 4	5·4	B5	B7	*ns*	−0·6	·006	−0·2	·008
211838	ϱ Aqr.	14·9	− 8 19	5·4	B8	B7	*ns*	..	..	−0·2	·008
212222	− Lac.	17·6	+41 36	6·3	B3	B4	*s*	−1·1	·003	−1·3	·003
222109	− And.	23 32·7	+43 52	5·9	B9	B9	*ns*	..	..	0·0	·007

Notes on Table III.

12301 : *c*B6*s* (W), *c*A0*s* (Ab) ; ·005 (W), ·009 (Ab).
18883 : B8*s* (W) ; ·006 (W).
24504 : B4*n* (W), B8σ + (U) ; ·009 (g), ·007 (W), ·006 (U).
24640 : B2*n* (W) ; ·005 (W).
58661 : A0τ − (U) ; ·003 (U).
87901 : B9*n* (W) ; ·058 (t), ·069 (W).
89774 : A0μ (U) ; ·008 (U).
139892 : B6*n* (W) ; ·009 (W).
142926 : A0σ (U) ; ·008 (U).
144470 : π = ·009 (g).
145389 : A0σ (U) ; ·014 (t), ·014 (U).
145502 : B2*n* (W) ; ·009 (g), ·021 (t), ·007 (W).
146926 : B9*n* (W), B8*n* (Ab) ; ·010 (W), ·009 (Ab).
179406 : B3*n* (W) ; ·005 (g), ·005 (W).
187879 : Bτ − (U) ; ·003 (U).
194636 : B8*s* (W) ; ·007 (W).
196775 : B4*n* (W) ; ·005 (W).
197392 : B8*s* (W) ; ·006 (W).
198667 : B8*s* (Ab) ; ·008 (Ab).
199478 : *c*B8*s* (W) ; ·005 (W).
210424 : B7*s* (W) ; ·006 (W).

TABLE IV.

Peculiar Stars.

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M_3.	10. π_3.
24479	– Cam.	3 48·6	+62 47	4·9	B9	B8	*ns*	−0·1	·010
30614	*α* Cam.	4 44·1	+66 10	4·4	Bo	Bo	*ns*	−3·4	·003
32549	11 Ori.	4 58·9	+15 16	4·7	B9	B9	*ns*	0·0	·012
37202	*ζ* Tau.	5 31·7	+21 5	3·0	B3*p*	B4	*nn*	−0·5	·020
111133	– Vir.	12 42·0	+ 6 30	6·4	B9	B9	*s*	−0·3	·005
142983	48 Lib.	15 52·6	−13 59	4·7	B3*p*	B5	*n*	−0·5	·009
203467	6 Cep.	21 17·3	+64 27	5·2	B3*p*	B3	*n*	−1·1	·006
212076	31 Peg.	22 16·6	+11 42	4·9	B3*p*	B3	*nn*	−0·8	·007
212593	4 Lac.	22 20·4	+48 58	4·6	B8*p*	B8	*ss*	−1·3	·007

Notes on Table IV.

24479: B9*n* (W), A0*σ* – (U); ·013 (W), ·011 (U). The lines 4481 and 4472 are *s*; remainder *ns* or *n*.

30614: Bo*s* (W); – ·025 (t), ·004 (W). Wedge measures give discordant results from different He lines, and all lines give rather faint abs. mag. Mean $M_1 = -2{\cdot}0$, $\pi_1 = {\cdot}005$.

32549: Numerous metallic lines visible, but [K] weak.

37202: B4*n* (W); – ·001 (t), ·019 (W). H lines *ss*, remainder *nn*. Sharpness of H lines probably due to masking of true *nn* lines by superposed double emission.

111133: Abnormally strong Sr^+, Fe^+, and other metallic lines. [K] invisible. *Cf.* H.D. 32549.

142983: H lines *s*, He lines *n*. *Cf.* H.D. 37202. Same Cause ?

203467: H lines broad, but sharp edges. He lines *ns*. Bright H*β*.

212076: H lines *nn*, He lines *ns*. Spectrum variable. In 1923: H*β* bright central emission on *nn* absorption. He bordered with emission on violet side. In 1925: H emission lines visible at least as far as H_ϵ. Other emission lines doubtful.

212593: One negative shows He lines *nn*. Other negatives normal; [K] strong and sharp.

Faint Stars in the Selected Areas with large Proper Motion. Third List from the Radcliffe Observatory, Oxford. By H. Knox-Shaw, M.A., D.Sc., Radcliffe Observer.

This list is uniform with the two earlier ones published in *M.N.*, **86**, 342, and **87**, 225, to the former of which reference should be made for explanation. The present list contains nineteen stars with proper motions as large as 0″·1 per annum from fifteen areas. At the end of the table are also given some particulars of two stars in Area 62. The reduction of this area is not yet complete, but it is evident that

the two stars have a common proper motion. That of the bright star, B.D. + 29° 3165 is given in Schorr's *Eigenbewegungs-Lexikon* as − 0″·13, + 0″·17.

The proper motions contained in this and the two former lists are subject to corrections for magnitude-equation,* which in the case of the brighter stars may amount to several hundredths of a second per annum. The plates were measured and reduced under the superintendence of Mr. H. G. Scott Barrett.

Selected Area No.	Area measured.	No. of Stars measured.	No. of Comparison Stars.	Mean Mag. of Comparison Stars.	Limiting Magnitude.	Star No.	Magnitude.	R.A. 1900·0.	Dec. 1900·0.	Annual Proper Motion. μ_α	μ_δ	μ
								h m s	° ′	″	″	″
4	50′×50′	279	108	12·8	15·1	26	11·3	8 0 24	+74 28·0	−0·066	−0·260	0·268
40	40×40	386	122	12·5	14·9	—						
42	40×40	444	140	13·4	15·3	—						
46	50×50	263	81	12·7	15·4	—						
51	50×50	396	140	13·0	15·2	—						
55	60×60	159	79	13·6	15·3	303	14·4	11 29 46	+30 28·2	− ·102	·000	·102
						169	9·5	11 29 48	+29 58·7	+ ·133	− ·116	·176
						33	14·5	11 30 52	+29 24·0	+ ·285	− ·832	·879
						318	12·3	11 31 51	+30 21·3	− ·140	− ·013	·141
65	40×40	339	214	13·3	15·0	—						
68	60×60	194	78	12·7	15·1	179	10·3	0 10 13	+15 15·0	+ ·110	+ ·020	·112
						384	10·5	0 10 54	+15 42·8	− ·003	− ·162	·162
						335	9·7	0 12 24	+15 33·2	− ·032	− ·122	·126
						398	7·2	0 12 38	+15 46·6	+ ·236	− ·060	·244
74	40×40	406	172	13·2	15·0	150	8·6	6 16 37	+14 54·8	+ ·024	− ·126	·128
82	60×60	104	48	12·9	14·6	375	14·0	14 15 57	+15 44·2	− ·086	+ ·070	·111
84	60×60	238	91	12·8	14·9	636	10·9	16 13 24	+15 28·2	− ·104	+ ·117	·157
88	40×40	324	164	12·9	14·5	559	11·3	20 11 4	+15 3·3	+ ·009	− ·184	·184
90	50×50	243	130	13·0	15·0	251	14·3	22 11 8	+15 3·5	+ ·152	− ·024	·154
						166	14·2	22 12 27	+14 51·7	+ ·202	− ·062	·211
						110	10·6	22 14 27	+14 49·1	+ ·158	− ·066	·171
92	60×60	134	60	12·7	14·9	409	11·2	0 50 6	+ 0 23·6	− ·236	− ·123	·266
106	60×60	159	61	12·8	14·2	432	13·0	14 37 37	− 0 17·2	− ·101	+ ·009	·101
						1124	11·2	14 38 9	+ 0 23·9	− ·098	− ·049	·110
62	50×50					221	7·6	17 56 48	+29 34·3			
						226	13·4	17 56 55	+29 33·9			

* *M.N.*, **87**, 439.

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

The following measures have been made with the 8-in. refractor lent by the Society:—

Star.	1925. R.A. h m	Dec. ° ′	Mags.	Position Angle. °	Distance. ″	No. of Nights.	Date 1900+.
OΣ 12 (λ Cas)	0 27·6	+54 7	5·6, 5·9	165·0	0·53	3	27·93
Σ 60 (η Cas)	0 44·4	57 26	4·0, 7·6	267·0	8·15	3	26·95
				268·1	8·14	3	27·90
Σ 174 (1 Ari)	1 46·0	21 54	6·2, 7·4	166·0	2·94	3	26·87
Σ 186	1 52·0	1 28	7·2, 7·2	41·6	1·36	3	26·87
OΣ 38 (γ^2 And)	1 59·3	41 58	5·0, 6·2	103·7	0·51	5	26·08
				105·6	0·49	3	27·06
Σ 795 (52 Ori)	5 43·9	6 26	6·2, 6·2	27·4	1·43	3	27·17
Σ 861	6 6·5	30 42	8·2, 8·2	317·6	1·69	3	26·09
Σ 1110 (Castor)	7 29·8	32 3	2·7, 3·7	212·6	4·58	11	26·18
				211·4	4·47	12	27·19
Σ 1126	7 36·1	5 24	7·2, 7·5	149·5	1·13	3	27·07
Σ 1196 (ζ Cnc) AB	8 7·9	17 53	5·0, 5·7	228·4	0·60	4	26·16
				216·0	0·52	5	27·13
Σ 1273 (ε Hya) AB, C	8 42·8	6 42	3·8, 7·8	247·1	3·07	3	26·13
Σ 1322	9 8·5	16 50	7·7, 8·2	54·0	1·64	3	26·13
Σ 1348	9 20·5	6 40	7·5, 7·6	319·6	2·10	3	27·12
Σ 1356 (ω Leo)	9 24·4	9 23	6·2, 7·0	132·7	1·05	3	27·09
Σ 1355	9 32·3	6 34	7·2, 7·2	337·2	2·62	3	26·15
				336·8	2·73	3	27·12
Σ 1374	9 36·7	39 17	7·0, 8·3	290·3	3·21	5	27·14
OΣ 229	10 43·7	+41 31	6·7, 7·1	306·6	0·62	3	27·19
Σ 1500	10 56·2	− 3 5	7·6, 8·2	306·7	1·36	3	27·31
Σ 1504	11 0·1	+ 4 2	7·5, 7·6	291·6	1·19	3	26·15
Σ 1523 (ξ UMa)	11 14·2	31 58	4·0, 4·9	79·5	1·99	6	27·10
Σ 1527	11 15·1	14 41	6·9, 8·1	19·3	2·96	3	27·33
Σ 1536 (ι Leo)	11 20·0	10 57	3·9, 7·1	25·4	1·11	4	27·49
Σ 1639	12 20·6	26 0	6·7, 7·9	337·5	0·73	3	27·30
Σ 1647	12 26·7	+10 8	7·5, 7·8	229·6	1·47	3	26·19
Σ 1670 (γ Vir)	12 37·9	− 1 2	3·0, 3·0	321·3	5·90	5	26·36
Σ 1734	13 16·9	+ 3 20	7·2, 7·9	185·4	1·13	3	26·20
Σ 1770	13 34·7	51 6	6·4, 7·9	121·0	1·91	3	26·09
Σ 1816	14 10·6	29 27	7·0, 7·1	79·6	1·00	3	27·21
Σ 1865 (ζ Boo)	14 37·6	14 3	3·5, 3·9	135·5	0·94	5	26·25
				135·4	0·97	6	27·19
Σ 1867	14 37·6	31 36	7·7, 8·2	11·1	0·82	3	27·27
Σ 1871	14 38·9	51 42	7·0, 7·0	297·2	1·90	3	26·28

Star.	1925. R.A. h m	1925. Dec. ° ′	Mags.	Position Angle. °	Distance. ″	No. of Nights.	Date 1900 +.
Σ 1879	14 42·5	9 58	7·8, 8·8	109·1	0·80	3	26·33
Σ 1890 (39 Boo)	14 47·2	49 2	5·8, 6·5	222·7	3·07	3	27·22
Σ 1888 (ξ Boo)	14 47·9	19 24	4·7, 6·6	38·5	3·26	5	26·28
				33·8	3·24	5	27·21
OΣ 287	14 48·7	45 13	7·5, 7·6	149·4	0·80	3	26·23
OΣ 288	14 49·9	16 0	6·4, 7·1	184·3	1·69	4	26·26
				184·0	1·44	5	27·24
Σ 1926	15 12·1	38 34	6·1, 8·4	252·7	0·79	3	26·25
Σ 1937 (η Crb)	15 20·1	30 33	5·2, 5·7	163·0	0·52	4	26·46
Σ 1938 (μ^2 Boo)	15 21·7	37 36	6·7, 7·3	39·8	1·42	3	27·27
Σ 1954 (δ Ser)	15 31·2	10 47	3·0, 4·0	181·3	3·56	3	27·28
Σ 1988	15 53·2	12 42	7·5, 8·2	257·3	2·63	3	27·46
OΣ 303	15 57·4	13 29	7·4, 7·9	153·3	0·82	3	27·30
Σ 2015	16 6·5	+45 32	7·7, 8·8	159·3	2·83	3	26·30
Σ 1998 (ξ Sco)	16 9·3	−11 11	4·9, 5·2	190·2	1·19	4	27·33
Σ 2021	16 9·8	+13 44	6·7, 6·9	158·8	3·99	3	27·30
Σ 2023	16 10·8	5 43	8·0, 9·0	228·4	1·36	3	27·37
Σ 2027	16 11·5	4 27	8·2, 8·2	74·9	1·90	3	27·37
Σ 2047	16 21·0	47 47	7·5, 8·0	327·8	2·15	3	26·33
Σ 2054	16 22·8	61 51	5·7, 6·9	354·8	0·90	3	27·35
OΣ 312 (η Dra)	16 23·0	61 40	2·1, 8·1	140·4	5·24	3	26·28
Σ 2049	16 24·8	26 8	6·5, 7·5	203·2	0·97	4	27·32
Σ 2052	16 25·6	18 33	7·5, 7·5	86·7	0·61	3	26·35
Σ 2055 (λ Oph)	16 27·1	2 8	4·0, 6·1	97·1	0·59	2	26·40
OΣ 313	16 30·0	40 16	7·2, 7·8	141·8	0·70	3	27·34
Σ 2078 (17 Dra)	16 34·4	53 4	5·0, 6·0	109·5	3·31	3	27·34
Σ 2084 (ζ Her)	16 38·5	31 43	3·0, 6·5	59·3	1·47	3	26·34
				53·8	1·20	5	27·36
Σ 2091	16 39·6	41 19	7·5, 8·0	306·2	1·02	4	26·39
Σ 2094	16 41·0	23 38	7·3, 7·6	75·8	1·49	3	26·37
Σ 2107	16 48·9	28 47	6·5, 8·0	39·8	0·72	4	26·33
Σ 2114	16 58·4	8 33	6·2, 7·4	168·6	1·05	3	27·37
Σ 2168	17 24·0	35 49	7·5, 8·2	197·9	2·41	3	26·34
Σ 2180	17 27·1	50 47	6·2, 9·5	262·6	3·22	3	26·28
Σ 2272 (70 Oph)	18 1·6	2 33	4·1, 6·1	126·2	6·19	4	26·39
				124·6	6·08	5	27·50
Σ 2292	18 9·1	27 37	8·0, 8·1	264·5	0·96	3	27·44
Σ 2309	18 17·1	25 30	8·5, 9·0	351·8	3·63	3	27·46
Σ 2375	18 41·8	5 25	6·2, 6·6	115·2	2·42	3	27·44
Σ 2402	18 46·2	10 35	8·0, 8·4	205·0	0·97	3	27·45
Σ 2422	18 54·1	25 59	7·6, 7·7	87·0	0·55	3	27·48
Σ 2437	18 58·5	19 4	7·8, 8·0	49·8	0·59	2	26·49

Star.	1925. B.A. h m	Dec. ° ′	Mags.	Position Angle. °	Distance. ″	No. of Nights.	Date 1900+.
Σ 2455	19 3·7	22 3	7·2, 8·3	56·8	4·48	3	27·50
Σ 2583 (π Aql)	19 45·2	11 38	6·0, 6·8	112·0	1·53	3	27·66
OΣ 410	20 36·8	40 18	6·4, 6·7	10·7	0·61	2	27·80
Σ 2742	20 58·5	6 54	7·1, 7·1	218·8	2·68	2	27·79
Σ 2744	20 59·2	1 15	6·3, 7·0	150·3	1·27	2	27·78
Σ 2804	21 29·5	20 23	7·3, 8·0	338·9	2·99	3	27·62
Ho 164 AB	21 37·8	+34 44	8·0, 8·0	64·6	3·51	3	27·64
Σ 2909 (ζ Aqr)	22 25·0	− 0 24	4·0, 4·1	300·9	2·59	3	27·64

Headley Rectory, Epsom,
1928 *January* 9.

New Double Stars. By Rev. T. E. Espin and W. Milburn.

The measures of new double stars are arranged as in previous lists. An asterisk in column 1 denotes that additional information is given in the notes. Throughout the year the clear nights have been few and the definition generally poor.

Espin No.	B.D. °	R.A. (1900). h m	Dec. ° ′	P. °	D. ″	Mags.	Nts.	Date 1927.	
2274	+32·35	0 12·2	+32 30	166·0	2·16	9·4, 11·7	3	·952	
2275	+32·178	57·6	32 20	144·5	3·56	9·7, 10·7	2	·004	(1928)
2276	+34·498	2 36·0	34 19	54·8	5·12	9·5, 9·6	2	·098	BC
				46·0	31·30	A= 9·3	2	·098	AB
2277	+36·649	3 5·8	36 12	231·8	2·64	9·5, 9·7	2	·563	
2278	+38·960	4 46·7	38 14	201·0	3·87	9·5, 13·0	3	·717	
2279	..	6 3·8	37 45	285·6	5·33	9·4, 11·5	2	·098	
2280 *	+35·1817	8 18·1	34 54	262·4	3·74	9·4, 10·7	2	·250	
2281	..	9 8·7	34 25	59·3	2·87	10·0, 12·0	1	·241	
2282 *	+33·2034	10 42·1	33 16	330·0	4·16	9·2, 13·5	2	·317	
2283	+33·2059	53·4	33 29	104·7	5·29	9·5, 11·0	2	·272	
2284	+34·2228	11 27·0	33 56	73·9	2·94	9·5, 9·6	3	·286	
2285	+33·2134	32·8	33 20	341·5	9·35	9·5, 10·5	2	·317	
2286	+34·3123	18 3·4	34 39	172·4	5·49	9·5, 13·0	3	·642	
2287	..	45·6	32 54	287·4	2·87	9·6, 11·0	2	·652	
2288	+33·3829	19 4·0	33 53	58·4	4·47	9·5, 9·7	2	·713	
2289	+34·3460	9·2	34 17	199·4	2·06	9·5, 12·0	2	·652	
2290 *	+33·3373	9·5	33 50	329·2	2·58	9·3, 11·5	3	·721	
2291	+33·3399	13·3	33 47	211·9	2·96	9·5, 12·0	3	·748	
2292	+33·3398	13·5	33 21	259·4	5·79	9·1, 11·5	2	·755	
2293	+33·3418	16·0	33 53	88·6	6·50	9·3, 10·6	2	·701	
2294	+33·3420	16·3	33 47	50·5	2·64	9·4, 12·5	2	·713	

Espin No.	B.D.	R.A. (1900).	Dec.	P.	D.	Mags.	Nts.	Date 1927.	
	°	h m	° ′	°	″				
2295	+33·3485	27·3	33 46	202·2	4·18	9·0, 12·0	2	·762	
2296	+34·3611	30·4	34 25	126·1	3·60	9·3, 12·0	2	·652	
2297 *	+33·3525	33·7	33 19	180·0	7·66	8·7, 9·0	3	·821	AB
				283·2	20·23	C= 10·0	2	·834	AC
2298	+34·3663	37·5	34 13	317·5	7·74	9·2, 13·0	2	·685	
2299	+34·3762	48·2	34 34	125·4	6·74	9·5, 14·0	3	·721	
2300	+34·3764	48·4	34 34	252·2	3·70	9·5, 12·4	3	·721	
2301	+33·3650	51·3	33 59	286·8	3·29	9·5, 13·0	3	·705	
2302	+34·3959	20 13·9	34 59	350·5	3·30	9·3, 11·5	3	·661	
2303	+34·4002	20·4	34 57	22·6	4·97	9·2, 10·6	4	·711	
2304	+34·4052	26·1	34 34	45·7	3·16	9·2, 13·0	3	·748	
2305	+34·4054	26·7	34 32	288·4	6·47	9·2, 12·0	3	·748	
2306	+34·4072	29·0	34 55	326·2	6·85	9·2, 13·5	3	·661	
2307	+34·4090	31·6	34 29	84·0	4·14	9·3, 12·0	3	·680	
2308	+33·3965	32·2	34 5	237·5	7·28	9·4, 12·0	2	·734	
2309	+33·3995	38·4	33 18	162·4	4·60	9·4, 11·2	2	·862	
2310	+33·4000	39·1	33 49	34·8	4·77	9·3, 9·8	2	·839	
2311	+33·4014	40·7	33 35	38·0	7·30	9·2, 14·0	3	·833	
2312	+33·4091	52·1	33 12	17·6	3·10	9·5, 11·5	2	·846	
2313	+33·4171	21 3·7	33 57	183·5	6·68	9·5, 14·0	3	·705	
	(*h* 1614)	..	..	257·8	10·34	C= 10·5	3	·705	
	+33·4174	6·4	33 54	33·6	13·66	9·0, 10·0	2	·713	
2314	+33·4186	7·1	33 57	212·5	5·93	9·2, 9·4	3	·705	
2315	+33·4188	7·5	33 54	38·4	4·82	9·3, 11·5	2	·669	AB
				56·5	11·18	C= 13·0	2	·698	AC
2316	+32·4097	10·1	33 8	5·9	5·67	8·5, 9·6	3	·820	
2317	+33·4213	12·1	33 15	305·7	8·70	8·5, 13·2	2	·755	
2318	+32·4178	26·2	32 24	75·8	4·29	9·5, 11·0	2	·772	
2319	+31·4519	34·2	31 50	56·2	6·10	9·2, 13·0	3	·825	
2320	+32·4221	34·9	32 14	322·7	2·84	9·5, 12·0	3	·687	BC
				199·0	13·55	A= 9·2	3	·687	AB
2321	+32·4279	47·8	32 24	99·0	2·11	8·5, 11·5	3	·714	
2322 *	..	48·8	31 37	276·3	4·26	10·0, 11·5	2	·884	
2323	..	22 5·5	31 40	326·4	3·29	9·5, 13·0	2	·755	
2324	+31·4686	17·6	31 18	142·1	3·43	9·5, 9·6	2	·758	
2325	+31·4804	48·8	31 31	244·8	5·77	9·5, 9·5	2	·755	
2326	+32·4681	23 34·4	32 28	172·5	1·95	9·8, 12·0	4	·793	
2327	+31·4953	36·5	32 14	48·5	3·59	9·5, 9·7	2	·992	
2328	+32·4694	38·2	33 0	167·9	6·71	9·2, 11·2	3	·806	AB
				346·2	7·41	C= 11·0	3	·806	AC

Notes.

2280. S.P. of two.
2282. An 11 mag. at P. 252°, D. 26″, and a 10 mag., more distant at P. 318°.
2290. A 12 mag. at P. 73°, D. 17″, another more distant at P. 67°.
2297. An 11 mag., more distant than C., at P. 148°.
2322. Forms distant *comes* to B.D.+31°·4569, which star has at P. 80° an 11·5 nearer, with a 14 mag. S.F.

B.D. Stars Missing or Faint.

	R.A. (1855).	Dec.	Mags.		
	h m s	° ′			
B.D. +34·447	2 22 18·6	+34 29·2	9·3	1927 Feb. 4.	13 mag. E.
+33·360	25 30·9	+33 54·6	9·5	Nov. 27.	No star above 14 mag. here. E.
+27·620	3 54 1·4	+27 47·2	9·5	Feb. 8.	Decl. +27° 51′·2. M.
+38·961	4 43 24·4	+38 27·0	9·5	Feb. 8.	Decl. +38° 25′. E.
+34·2384	12 58 56·2	+34 50·2	9·5	Apr. 30.	No star above 12 mag. here. E.
+27·2282	13 38 31·7	+27 58·3	9·5	Apr. 30.	Faint. M.
+35·2996	18 26 48·2	+35 55·6	9·5	Aug. 17.	Not seen. E.
+34·3773	19 47 49·4	+34 34·7	9·5	Aug. 25.	Not seen. E.
+28·3554	52 45·9	+28 27·6	9·5	Sept. 9.	Missing. M.
+30·4625	21 4 26·3	+30 48·1	9·5	Nov. 16.	No star here. E.
+32·4172	23 11·7	+32 14·4	9·5	Oct. 6.	Not seen. E.
+27·4170	42 18·4	+27 43·2	9·5	Nov. 4.	No star here. M.
+30·4951	23 21 16·4	+30 57·1	9·5	Oct. 19.	No star here. E.

Milburn. No.	B.D.	R.A. (1900).	Dec.	P.	D.	Mags.	Nts.	Date 1927.
	°	h m	° ′	°	″			
511	..	0 14·2	+28 13	51·5	3·87	11·0, 12·0	2	·912
512	..	46·8	28 41	0·9	3·09	9·6, 13·2	3	·833
513	..	47·0	28 37	3·3	6·74	9·7, 10·5	3	·833
514	+28·230	1 18·3	28 27	277·8	2·60	9·2, 11·2	2	·018 (1928)
515	..	28·6	27 58	345·6	6·43	9·9, 10·8	3	·005 (1928)
516	+27·257	32·3	27 22	247·0	2·03	9·5, 10·2	2	·008 (1928)
517	..	2 18·7	29 4	38·5	4·98	9·6, 10·6	2	·549
518	..	3 21·6	28 8	254·4	3·10	9·6, 10·1	2	·098
519	+28·807	5 23·0	28 11	155·5	3·68	9·0, 11·2	2	·563
520	..	43·2	28 43	261·6	3·21	9·9, 10·7	3	·407
521	..	6 41·5	28 14	359·7	4·90	10·7, 10·8	3	·407
522	..	7 23·5	28 42	270·0	2·76	9·5, 12·5	2	·098
523	..	9 24·3	28 2	95·5	5·29	10·0, 10·5	1	·241
524	..	13 24·5	28 9	165·8	4·29	9·8, 10·3	1	·323
525	..	19 12·3	28 12	15·3	5·63	9·8, 10·8	2	·672
526	..	37·3	28 21	9·8	5·75	10·0, 11·0	2	·669
527	..	38·1	28 38	176·1	5·48	10·4, 10·7	2	·652
528	..	43·2	28 4	26·4	4·71	9·8, 10·5	3	·757
529	..	20 4·4	28 24	57·9	3·60	9·4, 10·0	2	·762
530	..	7·4	28 28	34·1	7·97	9·5, 10·0	2	·725
531	..	8·3	28 38	191·6	2·22	10·0, 11·0	3	·675
532	..	24·6	29 4	350·9	5·74	10·0, 10·2	2	·762

Milburn. No.	B.D.	R.A. (1900).	Dec.	P.	D.	Mags.	Nts.	Date 1927.	
	°	h m	° ′	°	″				
	+28·3771	25·6	28 17	330·5	10·70	9·5, 11·0	3	·735	
533	..	36·5	28 27	264·7	2·75	9·8, 10·5	2	·745	
534	..	45·4	28 1	139·2	4·33	9·8, 12·5	2	·834	
535	..	48·9	28 33	11·2	8·04	9·5, 10·0	2	·725	
536	..	21 1·2	28 37	235·6	4·67	10·5, 12·5	2	·713	
537	..	4·9	27 56	97·8	6·17	9·7, 10·0	2	·862	
	+29·4363	12·3	29 36	171·3	10·34	9·5, 10·0	3	·694	
538	..	12·6	29 36	311·6	3·33	10·2, 11·7	4	·699	
539	..	37·6	28 57	149·0	4·71	9·6, 10·2	2	·713	
540	..	41·7	27 49	252·0	3·14	11·0, 12·5	2	·758	AB
				323·8	8·00	C= 11·1	3	·770	AC
541	+27·4167	43·6	27 51	350·6	7·50	9·5, 13·5	2	·857	
542	..	22 4·5	27 42	270·6	3·24	12·0, 12·5	3	·851	
543 *	..	20·2	28 3	93·1	4·44	9·3, 10·5	2	·745	
544 *	..	27·5	28 15	88·6	3·87	9·5, 9·5	2	·778	
545	..	37·6	27 8	51·4	3·41	9·6, 12·8	3	·806	
546	..	44·4	26 59	259·7	2·79	10·0, 10·1	2	·754	
547	..	56·9	27 49	267·6	3·68	10·0, 10·5	2	·745	
548 *	+26·4605	23 15·1	26 38	279·0	6·16	9·4, 9·6	2	·857	
549	..	24·3	26 52	250·5	2·72	9·5, 13·0	3	·748	
550	..	30·5	27 43	185·5	3·22	9·4, 9·5	2	·794	
551	..	31·6	27 7	198·6	5·25	9·6, 10·2	2	·758	

Notes.

543. A 10·5 mag. at P. 210° ±, D. 15″ ±.
544. 11·0 mag. at P. 149° ±, D. 17″ ±, and 10·5 mag. at P. 184° ±, D. 22″ ±.
548. *Comes* P. 110° ±, D. 35″.

Pressures in the Calcium Chromosphere and Reversing Layer.

By E. A. Milne.

1. *Scope of the Paper.*—In a recent interesting paper Dr. A. Unsöld * has developed a theory of the solar chromosphere which leads to results markedly different from those obtained by the present writer. Taking account of the possibility of the dissociation of singly ionised calcium atoms into free electrons and doubly ionised calcium atoms, and calculating the pressure of radiation on the singly ionised atoms, he deduces that the calcium chromosphere consists, roughly speaking, of

* *Zeit. für Phys.*, **44**, 793, 1927.

a layer sharply bounded above at an approximately constant pressure of $1{\cdot}7 \times 10^{-7}$ atmospheres: more precisely, that as we descend the pressure rapidly increases, in a layer some 10 kms. or so thick, from some small pressure to a limiting pressure of $1{\cdot}7 \times 10^{-7}$ atmos. This pressure is much larger than that deduced by the present writer for the pressure at the base of the chromosphere, namely, 10^{-12} or 10^{-13} atmos.* Unsöld remarks that his estimate is in agreement with that deduced by Fowler and Milne for the pressure in a stellar atmosphere at which the H and K lines of Ca^+ originate, as shown by the position in the stellar temperature sequence at which these lines attain their maximum intensity. Unsöld observes that the discrepancy between my value 10^{-12} or 10^{-13} atmos. and his value $1{\cdot}7 \times 10^{-7}$ atmos. "is only to be expected, since, as we believe we have shown, the fundamental assumption of Milne's calculation is physically incorrect."

The fundamental assumption which Unsöld criticises is that I have assumed that the calcium chromosphere consists entirely of Ca^+ atoms and free electrons. Unsöld argues that a simple application of Saha's ionisation formula shows that this is unjustifiable. He mentions that He^+ (I.P. = 24·6 volts) makes itself apparent in the flash spectrum by the line $\lambda 4686$, and therefore Ca^{++} (I.P. = 11·82 volts) must also be present.

The broad answer to this objection is, first, that the chromosphere is not in thermodynamic equilibrium, and therefore Saha's ionisation formula is not applicable there; second, that when ionisations of Ca^+ atoms take place, as they will from time to time either by collisions or by absorption of radiation greater than $\lambda 1044$, the resulting Ca^{++} atom being nearly transparent to such radiation as is available will fail to be supported by radiation-pressure against gravity and will fall down through the chromosphere to the layer of higher density below. The actual calcium chromosphere must thus consist substantially of Ca^+ atoms and electrons. When any Ca^+ atom becomes ionised to Ca^{++}, there will be a slight lack of back radiation-pressure on the reversing layer from the chromosphere, and a new Ca^+ atom will be expelled to take the place of the one lost.

Presumably He^+ atoms remain because there is enough radiation-pressure to hold them up. In any case they appear only in the low-level chromosphere. Indeed neutral Ca atoms appear up to some distance (6000 kms., Mitchell; 2500 kms., Stratton and Davidson), though there should be very few present at $1{\cdot}7 \times 10^{-7}$ atmos. according to the ionisation formula. They too must undergo ionisation from time to time. As each Ca atom is ionised to Ca^+, a new Ca atom will be expelled from the reversing layer by the radiation in the line $\lambda 4227$, but to balance this a Ca^+ atom will fall back, as otherwise there would be one Ca^+ atom too many.

The purpose of this note is, first, to examine Unsöld's investigation in detail. We shall find that his conclusions are untenable. It will appear that what he has discussed is not the chromosphere but the lower-lying reversing layer. But, further, his density distribution is in error owing to an overestimate of selective radiation-pressure. We

* *M.N.*, **85**, 128, 1924.

therefore reinvestigate the equilibrium of the calcium reversing layer, and arrive at pressures in general agreement with the facts.

If we discuss as he does the condition of a layer of Ca, Ca^+, and Ca^{++} atoms and free electrons in thermodynamic equilibrium we readily find that at the top of the layer the density decreases rapidly, so that the layer has a comparatively sharp upper boundary. This is merely the well-known result that owing to the large value of solar gravity no extended high-level atmosphere is possible unless radiation-pressure almost exactly balances gravity. We now trace the density downwards. Unsöld adopts a value for radiation-pressure on Ca^+ atoms exceeding gravity, and uses the excess to support the associated Ca^{++} atoms in thermodynamic equilibrium. As a consequence he finds that the density rises to a limiting value corresponding to a pressure $1{\cdot}7 \times 10^{-7}$ atmosphere, and identifies this with the chromosphere. If it is the chromosphere, it must extend to the order of 10,000 kms. But a column 10,000 kms. high at a pressure of $1{\cdot}7 \times 10^{-7}$ atmos. is a physical impossibility: gas-pressure and radiation-pressure combined are totally inadequate to support such a column against solar gravity. Even a column 100 kms. thick at this pressure is an impossibility. Thus the layer whose equilibrium Unsöld is examining cannot be the solar chromosphere. A little consideration shows that what he is in fact discussing is not the chromosphere at all, but the high-level reversing layer. It is therefore only to be expected that the pressure he finds is in agreement with Fowler and Milne's reversing layer pressure for H and K.

If we now consider the state of affairs at the relatively sharply bounded upper limit of the layer under discussion, we see that there still remains an unbalanced radiation-pressure on individual Ca^+ atoms. They are therefore expelled and form a low-pressure screen which we identify with the chromosphere. The chromospheric pressure is the low value to which the reversing layer pressure * falls as we ascend, at the level at which thermodynamic equilibrium ceases to hold. The back radiation-pressure from the chromospheric screen maintains in equilibrium the Ca^+ atoms at the highest level of the reversing layer. Thus beginning with a layer in thermodynamic equilibrium, we are compelled afterwards to infer the existence of another layer, a low-pressure region, above it: these two layers we identify respectively with the reversing layer and chromosphere.

Unsöld's deduction that in the layer in thermodynamic equilibrium (wrongly identified by him with the chromosphere) the pressure tends to a constant limit as we descend is a consequence, first, of his high estimate of radiation-pressure; secondly, of the fact that in this layer he takes radiation-pressure to be constant. The numerical value of his estimate will be discussed later. It seems to me to be physically inadmissible. But in the layer in thermodynamic equilibrium radiation-pressure cannot be taken to be constant. In the chromosphere, which is in monochromatic radiative equilibrium, radiation-pressure is approximately constant because the net flux is constant. But in the

* More precisely, the partial pressure of Ca^+ atoms in the reversing layer. See § 3.

layer in thermodynamic equilibrium, as I have shown in a recent paper,* selective radiation-pressure decreases exponentially as we descend.

When we take a smaller estimate of selective radiation-pressure in the reversing layer (*i.e.* one less than gravity) we are not led to Unsöld's result of a limiting pressure. We shall see later that in this layer the pressure steadily increases downwards from the chromospheric value 10^{13} atmos. The layer in which Ca^+ atoms in thermodynamic equilibrium are abundant is of the order of 100 kms. in thickness. The relative concentration of Ca^+ atoms reaches a maximum, as we descend, at a pressure of calcium atoms of about 2×10^{-7} atmos.†

It has been mentioned that a column of the height of the chromosphere, say 10,000 kms., could not be supported if its pressure had Unsöld's value $1{\cdot}7 \times 10^{-7}$ atmos. As Unsöld used an equation of mechanical equilibrium, it may be asked how the discrepancy arises. The answer is that he has not traced inwards the intensities of the inward and outward streams of radiation implied by his distribution of density. A column of any length can of course be supported if the intensity incident on it at its base is sufficiently large. But on the sun this cannot exceed the photospheric value. As we go inwards, in radiative equilibrium (as assumed by Unsöld), though the difference between the two streams remains constant and equal to the net flux, each increases according to a linear function of the optical thickness. For a layer of height 10,000 kms. at Unsöld's pressure to be supported, the stream of radiation at the base would have to be some 10^4 times the photospheric value.

We now proceed to details.

2. *Impossibility of a Chromosphere at* $1{\cdot}7 \times 10^{-7}$ *Atmos.*—Let us consider a column of Ca^+ atoms and electrons, at a pressure of $1{\cdot}7 \times 10^{-7}$ atmos., 10,000 kms. in length, the latter being the observed order of magnitude of the thickness of the calcium chromosphere. If the solar effective temperature is $T_1 = 5740°$, as adopted by Unsöld, the boundary temperature will have the Schwarzschild value $T_0 = 5740 \times 2^{-\frac{1}{4}} = 4830°$. We will suppose the column to be at this temperature. If N is the number of calcium atoms per cm.^{3}, the pressure p is given by $p = NkT_0$. The number of atoms in the column is then 10^9 N, which, with $k = 1{\cdot}37 \times 10^{-16}$, $p = 0{\cdot}17$ dyne cm.$^{-2}$, comes to $1{\cdot}28 \times 10^{20}$ atoms. Each atom having a mass $40 \times 1{\cdot}64 \times 10^{-24}$ gram, the total mass is

$$8{\cdot}4 \times 10^{-3} \text{ gram} \quad . \quad . \quad . \quad . \quad (1)$$

Since at the sun's surface the value of gravity is $g = 2{\cdot}74 \times 10^4$, the gas-pressure 0·17 dyne cm.$^{-2}$ can support a mass

$$\frac{0{\cdot}17}{2{\cdot}74 \times 10^4} = 6{\cdot}2 \times 10^{-6} \text{ gram} \quad . \quad . \quad . \quad (2)$$

Thus gas-pressure at the base is quite inadequate to support the

* "Selective Radiation-Pressure and the Structure of a Stellar Atmosphere," *M.N.*, **87**, 697, 1927.

† The agreement of this figure with Unsöld's figure $1{\cdot}7 \times 10^{-7}$ atmos. is fortuitous, as our estimate depends on the I.P. of Ca as well as of Ca^+.

column, and we turn to radiation-pressure. Let $\pi I_\nu \Delta\nu$ be the outward flux in ν-radiation at the sun's surface, which we take to be the base of the column in question. Then

$$I_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT_1},$$

where $T_1 = 5740°$. Let $\pi F_\nu \Delta\nu$ be the residual flux in the centre of the H-K absorption line. We put $F_\nu = I_\nu/r$, where, according to Schwarzschild's measure,* adopted by Unsöld, $r = 11$. Then the flux $\pi(I_\nu - F_\nu)\Delta\nu$ must be returned by the layer to the photosphere. The momentum entering the layer per second is $\pi I_\nu \Delta\nu/c$, that leaving in the outward direction is $\pi F_\nu \Delta\nu/c$, and the recoil momentum arising from the returned radiation is $\pi(I_\nu - F_\nu)\Delta\nu/c$. Thus the total momentum acquired per second is

$$2\pi(I_\nu - F_\nu)\Delta\nu/c$$

$$= \left(1 - \frac{1}{r}\right)\frac{2\pi}{c}\,\frac{2h\nu^3}{c^2} e^{-h\nu/kT_1}\Delta\nu \quad . \quad . \quad . \quad (3)$$

Let $\Delta\nu$ correspond to a breadth $\Delta\lambda = 1$ Å. Then with $\lambda = 3950$ Å. (the mean of H and K), (3) comes to

$$4{\cdot}12 \times 10^{-4} \text{ gram cm. sec.}^{-2},$$

which can support a mass

$$\frac{4{\cdot}12 \times 10^{-4}}{2{\cdot}74 \times 10^{4}} = 1{\cdot}50 \times 10^{-8} \text{ gram} \quad . \quad . \quad (4)$$

If we multiply by 2 for the two H and K lines, and by 10 for their width, thus enormously exaggerating the amount of radiation absorbed and attributing the whole width of the lines to the chromosphere, we still get only 3×10^{-7} gram. Comparison with (1) shows that this is totally inadequate. The photospheric radiation would have to be at least 10^4 times as strong as it is to support a column 10,000 kms. long at Unsöld's pressure. Conversely, the actual photospheric radiation-pressure can at most support about 1 km. at Unsöld's pressure: and the gas-pressure itself could support only 10 kms. It follows that the layer Unsöld discusses cannot be the solar chromosphere.

If, on the other hand, we calculate the pressure a column 10,000 kms. long must have if it is to be supported by radiation-pressure, we find (1) and (4) that the pressure is

$$0{\cdot}17 \times \frac{1{\cdot}50 \times 10^{-8}}{8{\cdot}4 \times 10^{-3}} = 3 \times 10^{-7} \text{ dyne cm.}^{-2} = 3 \times 10^{-13} \text{ atmos.} \quad (5)$$

This is simply my original estimate of the chromospheric pressure. Actually it is an overestimate, as we have attributed to the chromosphere the whole depth of the H and K lines. As I have shown in my paper "Selective Radiation-Pressure and the Structure of a Stellar

* Revised measures have recently been given by Minnaert, *Zeit. für Phys.*, **45**, 610, 1927.

Atmosphere," the layer in local thermodynamic equilibrium below the chromosphere will itself cause a line with a residual intensity corresponding to the Schwarzschild boundary temperature $T_0 = 2^{-\frac{1}{4}}T_1$, *i.e.* a residual intensity $I_\nu(T_0)$. The above estimate therefore requires to be reduced by a factor

$$\frac{I_\nu(T_0) - F_\nu}{I_\nu(T_1) - F_\nu} = \frac{e^{-\frac{h\nu}{kT_1}\left(\frac{T_1}{T_0} - 1\right)} - r^{-1}}{1 - r^{-1}}.$$

Since $h\nu/kT_1 = 6{\cdot}32$, $T_1/T_0 = 1{\cdot}19$, $r = 11$, this factor comes to $0{\cdot}23$. The pressure is therefore about 10^{-13} atmos., and we attribute this pressure to the chromosphere.*

3. *Equilibrium of a Layer of Calcium Vapour in Thermodynamic Equilibrium.*—The following investigation is the kernel of the present paper. It is a natural continuation of my paper on " Selective Radiation-Pressure, etc.," taking account, as Unsöld does, of the varying ionisation from level to level, which in the paper mentioned was specifically neglected so as to emphasise the general principles. The conclusions are very different from Unsöld's.

Let now $\pi F_\nu \Delta\nu$ be the net flux in ν-radiation at any level, α_ν the atomic absorption coefficient, m' the mass of the atom. Then the force on the atom due to radiation-pressure is

$$\pi\alpha_\nu F_\nu \Delta\nu/c,$$

and the ratio of this to gravity which we shall denote by λ is

$$\lambda = \frac{\pi\alpha_\nu F_\nu \Delta\nu}{cm'g} \quad . \quad . \quad . \quad . \quad (6)$$

At the base of a fully supported chromosphere λ is approximately unity. It is unity throughout the whole of the chromosphere, since the latter is in monochromatic radiative equilibrium and the net flux there is constant. But as shown in " Selective Radiation-Pressure, etc.," in the layer in local thermodynamic equilibrium F_ν decreases inwards exponentially † with the optical thickness measured for ν-radiation. Consequently so does λ. The important point is that λ is not constant.

We allow for the presence of Ca, Ca^+, and Ca^{++} atoms. Suppose there are N calcium atoms per cm.^{3}, of which the fraction x are once ionised and the fraction y twice ionised. Then assuming thermodynamic

* This estimate depends of course on an assumed $\Delta\lambda = 1$Å. The core of the line, for which alone the high-level chromosphere can be responsible, is likely to be much narrower than this, but it seems better to have a definite number introduced into the calculations. A less crude theory would require some easily-made alterations.

† In monochromatic radiative equilibrium, the inward and outward streams increase at the same rate as we go inwards. But in local thermodynamic equilibrium the inward stream rises rapidly owing to the back radiation of the selectively absorbing atoms, whilst the outward stream rises at a slower rate determined largely by the temperature. Hence the difference F_ν decreases inwards.

equilibrium at temperature T, the Saha equations of first and second stage ionisation are

$$\frac{Nx(x+2y)}{1-x-y} = \frac{(2\pi m)^{\frac{3}{2}}}{h^3}(kT)^{\frac{3}{2}}e^{-\chi_1/kT} = K_1 \quad . \quad . \quad (7)$$

$$\frac{Ny(x+2y)}{x} = \frac{(2\pi m)^{\frac{3}{2}}}{h^3}(kT)^{\frac{3}{2}}e^{-\chi_2/kT} = K_2 \quad . \quad . \quad (8)$$

Here χ_1 and χ_2 are the first and second-stage I.P.'s of calcium 6·08 and 11·82 volts, and m is the mass of the electron. "Weight" factors are ignored. The partial pressure of free electrons is $N(x+2y)kT$, and the total pressure is $N(1+x+2y)kT$.

The question arises, What value are we to choose for T? The layer is traversed by radiation of temperature T_1, the sun's effective temperature, but on one side only. The atoms are probably themselves close to the Schwarzschild boundary* temperature $T_0 = 2^{-\frac{1}{4}}T_1$, and this is the temperature which will govern ionisation by collision. All things considered, it seems best to adopt the Schwarzschild value.† With $T_1 = 5740°$, we have for T the value $T = T_0 = 4830°$. The values of K_1 and K_2 are then found to be given by

$$kTK_1 = 2{\cdot}28 \times 10^2 \text{ dyne cm.}^{-2},$$
$$kTK_2 = 2{\cdot}23 \times 10^{-4} \text{ dyne cm.}^{-2}.$$

Thus K_2 is small compared with K_1. It follows that when NkT is much smaller than $2{\cdot}23 \times 10^{-4}$ dyne cm.$^{-2}$ $= 2{\cdot}23 \times 10^{-10}$ atmos., y is of the order of unity and x is very small. As we descend, *i.e.* as NkT increases, y decreases to almost zero and x increases to almost unity: at higher pressures still, x decreases again.

We first calculate the pressure at which x attains its maximum. Differentiating (7) and (8) we have

$$\frac{dx}{x} + \frac{dx+2dy}{x+2y} + \frac{dx+dy}{1-n-y} = -\frac{dN}{N},$$

$$\frac{dy}{y} + \frac{dx+2dy}{x+2y} - \frac{dx}{x} = -\frac{dN}{N}.$$

Putting $dx/dN = 0$ and eliminating dy/dN we find

$$\frac{2-x}{1-x-y} = \frac{x+4y}{y} \quad . \quad . \quad . \quad . \quad (9)$$

Since x is approximately unity here, we put $x = 1 - x'$ and treat y and x' as small. Solving (7), (8), and (9) for x', y, and N we find then

$$x' - y = \frac{N}{K_1}, \qquad y = \frac{K_2}{N}, \qquad x' - y = y,$$

* More accurately, $T_0 = (7/16)^{\frac{1}{4}}T_1$.

† It is to be remarked that we ignore the temperature gradient that must exist through the reversing layer itself, and take a mean value for T.

whence at the maximum of x

$$N = (K_1K_2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (10)$$

$$x_{max} = 1 - 2(K_2/K_1)^{\frac{1}{2}}, \quad y = (K_2/K_1)^{\frac{1}{2}} \quad . \quad (11), (12)$$

From (10) we find that x is a maximum where

$$NkT = 2{\cdot}25 \times 10^{-1} \text{ dyne cm.}^{-2} = 2{\cdot}25 \times 10^{-7} \text{ atmos.} \quad . \quad (13)$$

Let us use h to denote the depth measured inwards, and denote by h_1 the value of h where x is a maximum. We see that, practically speaking, above $h = h_1$ the gas is almost entirely Ca^{++} and Ca^+, below $h = h_1$ almost entirely Ca^+ and Ca. At $h = h_1$ itself, the gas is almost entirely Ca^+.

The equation of mechanical equilibrium is *

$$kT\frac{d}{dh}[N(1 + x + 2y)] = Nm'g[1 - \lambda x - \lambda'(1 - x - y)] \, . \quad (14)$$

where λ refers to the selective pressure of radiation on Ca^+ atom, (H and K lines) and λ' to the selective pressure on Ca atoms (g lines $\lambda 4227$). We specifically neglect the general pressure of radiation for simplicity's sake.

When x and y have been determined in terms of M from (7) and (8), (14) is a differential equation for N as a function of h. It is a sufficient approximation, however, to divide the range of integration into two parts as follows. For $h<h_1$, we assume $x + y = 1$ and apply (8): for $h>h_1$ we assume $y = 0$ and apply (7). Let h_0 be the upper boundary of the layer in thermodynamic equilibrium, and let x there have the value x_0: x_0 is the degree of ionisation of Ca^+ atoms *in thermodynamic equilibrium* at the pressure of the chromosphere, which is not to be confused with the degree of ionisation in the chromosphere itself.

For $h_0<h<h_1$, we have

$$\frac{m'g}{kT}(h - h_0) = \int_{x_0}^{x}\frac{d[N(3 - x)]}{N(1 - x\lambda)} = \int_{x_0}^{x}\frac{(1 - x)(2 - x)}{x(1 - \lambda x)}d\left(\frac{(3 - x)x}{(1 - x)(2 - x)}\right) \quad (15)$$

on substituting for N from (8). For $h>h_1$ we have

$$\frac{m'g}{kT}(h - h_1) = \int_{x_{max}}^{x}\frac{d[N(1 + x)]}{N[1 - \lambda x - \lambda'(1 - x)]}$$

$$= \int_{x_{max}}^{x}\frac{x^2}{(1 - x)[1 - \lambda x - \lambda'(1 - x)]}d\left(\frac{(1 - x)(1 + x)}{x^2}\right) \quad (16)$$

* This assumes that selective radiation-pressure is shared out amongst all the atoms, whether they be Ca, Ca^+, or Ca^{++}. It is quite possible that the selective pressure separates the atoms out, or tends to do so, just as in Dalton's law for isothermal atmospheres "selective weight" separates out the atoms of different masses. For an isothermal column, "selective weight" does not interfere with dissociative equilibrium ("Dissociative Equilibrium in an External Field of Force," *Proc. Camb. Phil. Soc.*, 22, 493, 1925), but the same problem for selective radiation-pressure has not yet been investigated.

on substituting for N from (7). The value of h_1 is given by

$$\frac{m'g}{kT}(h_1 - h_0) = \int_{x_0}^{x_{\max}} \frac{(1-x)(2-x)}{x(1-\lambda x)} d\left(\frac{(3-x)x}{(1-x)(2-x)}\right) . \quad (17)$$

The partial pressure of Ca^+ atoms at the top of the layer in thermodynamic equilibrium must be continuous with that at the base of the chromosphere. This is $x_0 NkT$. We have seen that the *total* pressure at the base of the chromosphere is 10^{-7} dyne cm.$^{-2}$. Thus

$$2x_0 N_0 kT = 10^{-7} \text{ dyne cm.}^{-2} . \quad (18)$$

But from (8),

$$kTN_0 \frac{(1-x_0)(2-x_0)}{x_0} = kTK_2 = 2{\cdot}23 \times 10^{-4} \text{ dyne cm.}^{-2} \quad (19)$$

Hence

$$\frac{2x_0^2}{(1-x_0)(2-x_0)} = 4{\cdot}48 \times 10^{-4},$$

whence

$$x_0 = 2{\cdot}1 \times 10^{-2} . \quad (20)$$

The integral in (17) cannot be evaluated exactly since λ decreases exponentially inwards with the optical thickness. But it is clear that the principal contributions are from the neighbourhoods of the poles $x = 0$ and $x = 1$. At $x = 0$, we have $\lambda = 1$: at $x = 1 - 2(K_2/K_1)^{\frac{1}{2}}$, $h = h_1$, let $\lambda = \lambda_1$. Then we have approximately

$$\begin{aligned} \frac{m'g}{kT}(h_1 - h_0) &\sim 3\int_{x_0}^{1} \frac{dx}{x} + \frac{2}{1-\lambda_1}\int_0^{x_{\max}} \frac{dx}{1-x} \\ &\sim -3 \log x_0 + \frac{1}{1-\lambda_1} \log \frac{1}{4}\frac{K_1}{K_2} \\ &\sim 11{\cdot}6 + \frac{1}{1-\lambda_1}\left[-1{\cdot}4 + \frac{\chi_2 - \chi_1}{kT}\right] . \quad (21) \end{aligned}$$

The value of $m'g/kT$ is $2{\cdot}72 \times 10^{-6}$: that of $(\chi_2 - \chi_1)/kT$ is 13·8. If for the sake of definiteness we take $\lambda_1 = \frac{1}{2}$, we arrive at

$$h_1 - h_0 = 130 \text{ kms.} . \quad (22)$$

A rough *a posteriori* evaluation of the optical thickness shows that more probably at $h = h_1$, λ has decayed nearly to 0. The value $\lambda_1 = 0$ gives

$$h_1 - h_0 = 90 \text{ kms.} . \quad (23)$$

Thus the fractional concentration of Ca^+ atoms rises to a maximum almost unity, some 100 kms. below the base of the chromosphere: and the pressure of calcium atoms here is about $2{\cdot}25 \times 10^{-7}$ atmos. Below this level the pressure increases and the fractional concentration of Ca^+ atoms decreases, giving place to Ca atoms. Throughout the whole of the layer, in which the composition changes from almost entirely Ca^{++} atoms, through almost entirely Ca^+ atoms down to a mixture of Ca^+ and Ca atoms, the equilibrium is of the gravitational "partly supported" type. At still greater depths, of course, increasing tem-

perature will cause ionisation of the Ca atoms and Ca^+ atoms in turn. (This inversion of the relative ionisation with depth was first pointed out by Pannekoek.)

This picture of the layer in local thermodynamic equilibrium is quite consistent with an almost entirely Ca^+ upper chromosphere, and a Ca^+ and Ca lower chromosphere. For these atoms are projected out of the layer in thermodynamic equilibrium by the uncompensated selective radiation-pressures occurring at the top of the layer.

4. *Comparison with Unsöld.*—Unsöld's line of argument amounts to the following. From a theoretical evaluation of the Einstein probability coefficient $B_{1\to 2}$ and the observed residual intensity in the H and K lines, he calculated that $\lambda = 1{\cdot}64$, and takes it as constant. Since $\lambda > 1$, the integrand in (15) possesses a pole at $x = 1/\lambda$ and as h increases x approaches this limit, and the pressure accordingly also approaches a limit. Putting $y = 1 - x$, $x = 1/\lambda$ in (8), we find

$$N\!kT = \frac{\lambda}{(\lambda - 1)(2\lambda - 1)} k\mathrm{T}\mathrm{K}_2.$$

The total pressure is $N\!kT(1 + x + 2y) = N\!kT(3 - 1/\lambda)$, *i.e.*

$$k\mathrm{T}\mathrm{K}_2 \frac{3\lambda - 1}{(\lambda - 1)(2\lambda - 1)} = 2{\cdot}69 k\mathrm{T}\mathrm{K}_2. \quad . \quad . \quad (24)$$

He has adopted a temperature which makes $k\mathrm{T}\mathrm{K}_2 = 0{\cdot}035$ dyne cm.$^{-2}$. The limiting value of the total pressure is therefore 0·09 dyne cm.$^{-2}$, in sufficient agreement * with his value 0·17. The pressure according to Unsöld never rises above this value. With the value we have chosen for T Unsöld's limiting pressure would be 6×10^{-4} dyne cm.$^{-2}$.

It is clear that the structure of the layer as found by Unsöld depends entirely on the value of λ being greater than 1. He calculates λ from the formula

$$\lambda = \frac{h\nu \mathrm{B}_{1\to 2}}{2cmg} \cdot \frac{\frac{1}{2}\mathrm{I}_\nu}{r} \quad . \quad . \quad . \quad . \quad (25)$$

the relation which I have used for the chromosphere.† But this is a dangerous formula from which to attempt to calculate λ exactly and infer that it is greater than 1. For there is an uncertainty in r, an uncertainty in the value of T to use in I_ν, and an uncertainty in $\mathrm{B}_{1\to 2}$. For $\mathrm{B}_{1\to 2}$ Unsöld adopts Reiche and Thomas's value

$$\mathrm{B}_{1\to 2} = \frac{4\pi}{c} \cdot \frac{\pi e^2}{mh\nu} \quad . \quad . \quad . \quad . \quad (26)$$

We may ask whether this can indicate more than the order of magnitude of $B_{1\to 2}$, for it involves no specialisation to the structure of the Ca^+ atom, and the H and K lines, which must certainly influence its absorption coefficient. If we use this value of $\mathrm{B}_{1\to 2}$ to determine τ, the mean life

* Unsöld makes an approximation which we have not found necessary. He solves (8) on the assumption that x is approximately unity, but he later finds the limiting value of x, namely, 1/1·64, to be appreciably different from unity.

† In chromospheric investigations I have assumed $\lambda = 1$ for a "fully supported" atmosphere, and in effect deduced $\mathrm{B}_{1\to 2}$ from observation.

of the excited Ca^+ atom, we find $\tau = 0{\cdot}7 \times 10^{-8}$ sec. in good agreement with the value $\tau = 1{\cdot}8 \times 10^{-8}$ secs. which I have deduced from the residual intensity in the H and K lines, using (25) with $\lambda = 1$. This shows that (25) with $B_{1\to 2}$ given by (26) is correct as regards order of magnitude, but there still remains the possibility of uncertainties in r and I_ν.*

But a more important consideration is that physically a value of λ greater than unity is an impossibility in a steady state. The residual intensity in the line must adjust itself so as to give a value of $\lambda \leqslant 1$. If temporarily we suppose that at the top of the layer in local thermodynamic equilibrium λ has a value >1, then Ca^+ atoms are ejected, and the chromosphere grows until its back radiation-pressure maintains the individual Ca^+ atoms at its base in equilibrium, *i.e.* until $\lambda \leqslant 1$.

As I have pointed out before, we can see in a general way that λ must in fact be very close to unity at the base of the chromosphere. If λ there is appreciably less than unity, the chromosphere is heavily condensed to its base: it is indistinguishable as regards its density gradient from the layer in local thermodynamic equilibrium below it, and will probably reassume a measure of thermodynamic equilibrium itself. The residual intensity in the line will then return to that corresponding to the boundary temperature, namely, $I_\nu(T_0)$. But this in turn implies expulsion of Ca^+ atoms, because it gives a selective pressure greater than $m'g$: hence atoms are expelled again. The exact pressure at which collisions become so rare as to permit of monochromatic radiative equilibrium involves very delicate considerations, as explained in "Selective Radiation-Pressure, etc." We prefer to determine it from observation. The use of equation (25), with $\lambda = 1$, to determine $B_{1\to 2}$ from observation is a very different matter from Unsöld's use of it to determine λ.

Lastly, the consequences of $\lambda > 1$ as regards the structure of the layer seem wholly unacceptable. First, it implies streams of radiation enormously greater than the photospheric intensity, as we have seen. In our work this is avoided because the value of λ tends to zero, as explained. Secondly, it leaves no place for the fitting of the column on to the Ca/Ca^+ layer beneath. If a star was made entirely of calcium, Unsöld's analysis would still never permit of the pressure exceeding his limiting value.

5. *General Description of the Calcium Reversing Layer.*—In the theory which we have sketched in § 3, gas-pressure increases in about 100 km. from 10^{-13} atmos. to 10^{-7} atmos.: selective radiation-pressure decreases from $\lambda = 1$ to nearly $\lambda = 0$: the weight of the column is supported mainly by gas-pressure. These features fit very closely what is known of the sun's reversing layer. That the reversing layer pressure in stellar atmospheres is commonly taken to be 10^{-4} atmos. involves no inconsistency, for this value is determined from the behaviour of subordinate lines, which originate deeper down. As

* If (26) is exactly true, then $\lambda = 1$ implies a smaller residual intensity in the centre of the lines than is given by Schwarzschild's measures. It is quite possible for the lines to have a very narrow black core, which would be difficult to detect photometrically. Minnaert (*loc. cit.*) has in fact found the residual intensity in the K line to be 7 per cent., as compared with Schwarzschild's 9 per cent. This reduces Unsöld's value of λ from 1·64 to 1·28.

Unsöld mentions, the pressure found by Fowler and Milne for the layer in which the H and K lines originate was $0{\cdot}8 \times 10^{-7}$ atmos., and we see now that this is practically the pressure of the layer where the Ca^+ atoms in the sun reach their maximum relative concentration.

6. *Unsöld's Explanation of the width of Fraunhofer Lines.*—The low-pressure chromosphere can only be responsible for deepening the blackness of the central core of the Fraunhofer line. The wings of the line must be formed in the higher-pressure reversing layer beneath. Unsöld's explanation of the broad wings of H and K given in his recent paper is not fundamentally affected by our destructive criticism of his theory of the chromosphere. His explanation required a certain total number of atoms, namely, $4{\cdot}5 \times 10^{19}$ atoms per cm.^{2}. He found those in a column 14,000 km. thick at 5×10^{-8} atmos. We have seen that no such column can exist. To find these atoms we have only, however, to look lower down. The necessary atoms are simply at a higher pressure, and compressed to a correspondingly shorter column, than estimated by Unsöld.

The number of overlying Ca^+ atoms per cm.^{2} in the low-pressure chromosphere is negligible compared with $4{\cdot}5 \times 10^{19}$. We will therefore estimate the number in the layer in thermodynamic equilibrium. In the layer $h = h_0$ to $h = h_1$, the number is

$$\int_{h_0}^{h_1} \mathrm{N}x\,dh,$$

or, using (14) and (8) with $x + y = 1$,

$$\frac{k\mathrm{TK}_2}{m'g}\int_{x_0}^{x_{\max}} \frac{x}{1-\lambda x}\,d\left(\frac{x(3-x)}{(1-x)(2-x)}\right).$$

This is approximately

$$\frac{k\mathrm{TK}_2}{(1-\lambda_1)m'g}\log\frac{\mathrm{K}_1}{\mathrm{K}_2} = \frac{k\mathrm{TK}_2}{(1-\lambda_1)m'g}\cdot\frac{\chi_2-\chi_1}{k\mathrm{T}}.$$

This with $\lambda_1 = 0$ comes to $1{\cdot}7 \times 10^{15}$, which is again negligible compared with $4{\cdot}5 \times 10^{19}$. We must therefore look for these atoms in the layers below $h = h_1$. Suppose that $4{\cdot}5 \times 10^{19}$ atoms cm.$^{-2}$ are encountered down to the layer $h = h'$, at which $x = x'$. Then

$$4{\cdot}5 \times 10^{19} = \int_{h_1}^{h'} \mathrm{N}x\,dh,$$

or, using (14) and (7) with $y = 0$,

$$4{\cdot}5 \times 10^{19} = \frac{k\mathrm{TK}_1}{m'g}\int_{x_{\max}}^{x'} \frac{x}{1-\lambda x-\lambda'(1-x)}\,d\left(\frac{1-x^2}{x^2}\right).$$

We may here put $\lambda = \lambda' = 0$. We have then approximately

$$4{\cdot}5 \times 10^{19} = \frac{k\mathrm{TK}_1}{m'g}\cdot\frac{2(1-x')}{x'}.$$

From this we find $x' = 0{\cdot}85$. The corresponding value of $\mathrm{N}'k\mathrm{T}$ from

(7) is $N'kT = 47$ dyne cm.$^{-2}$, *i.e.* the pressure here is about 5×10^{-5} atmos. The depth below $h = h_1$ at which we encounter this is given, using (16), by

$$\frac{m'g}{kT}(h' - h_1) = \int_{x_{\max}}^{x'} \frac{x^2}{1-x} d\left(\frac{1-x'}{x^2}\right) = 2\left[\log \frac{1-x}{x}\right]_{1-2(K_2/K_1)^{\frac{1}{2}}}^{x'},$$

whence $h' - h = 30$ kms. Thus in the layer 30 kms. thick in which the pressure of calcium atoms increases from 2×10^{-7} atmos. to 5×10^{-5} atmos., there are $4{\cdot}5 \times 10^{19}$ atoms of Ca^+ per cm.^{2}, the number required by Unsöld. Of course the difficulty now is to explain why the Ca^+ atoms lower down still have no effect. Unsöld fitted his supposed chromospheric column straight on to the photosphere and did not enquire as to the effect of Ca^+ atoms in the photospheric layers. Thus the difficulty occurs in his work also. It arises because of the physically unjustifiable assumption of a definite radiating surface, the photosphere, with an atmosphere in front of it, which is implied by the simple Schuster theory. Actually any theory of absorption lines must take account of the merging of the layers in which the lines are formed into those in which the continuous spectrum is formed.*

7. *The Rate of Ionisation in the Ca^+ Chromosphere.*—Though the chromosphere is not in thermodynamic equilibrium, ionisation of the Ca^+ atoms will occur from time to time. In § 3 we have shown that if the chromosphere were in thermodynamic equilibrium and its pressure 10^{-13} atmos., only the fraction $x_0 = 0{\cdot}02$ of the atoms would be in the Ca^+ state, this being in fact the ratio of Ca^+ atoms to Ca^{++} atoms at the top of the reversing layer. But we cannot assume the high-level chromosphere to be in thermodynamic equilibrium. We cannot apply the Saha formula, for no Ca^{++} atom can be in mechanical equilibrium. Instead we have to calculate the actual number of ionisations per second.

It is easily seen that the rate of ionisation by collision at this pressure must be negligible. Any ionisations which occur must therefore be due to the absorption of radiation of wave-length shorter than corresponds to the ionisation potential $\chi_2 = 11{\cdot}82$ volts, or λ 1044 Å. Unfortunately we have no knowledge of the solar radiation in this region. It is certain that it must be much less than corresponds to black-body radiation, owing to molecular scattering. We will, however, attempt to estimate what the rate of ionisation would be if the sun radiates like a black body in this region.

Let $\psi(\nu)I_\nu d\nu dt$ be the probability that an atom is ionised in dt when exposed to isotropic radiation of intensity I_ν. Then by Kramers' formula,† apart from a factor of the order of unity, we have for the ionisation of a nucleus of atomic number Z with one electron (I.P. $= \chi_0$)

$$\psi(\nu) = \frac{16\pi^2}{3\sqrt{3}} \frac{Z^2e^6}{ch} \frac{\chi_0}{(h\nu)^3} \frac{4\pi}{h\nu}.$$

* A theory of Fraunhofer lines substantially equivalent to Unsöld's, as Unsöld mentions, was put forward by J. Q. Stewart, *Astrophys. Journ.*, **59**, 35, 1924. It appears to me to be subject to the criticisms given in *M.N.*, **85**, 739, 1925.

† The deduction of this formula is given in *M.N.*, **85**, 754, 1925, equations (4) and (15), combined with *Phil. Mag.*, **47**, 209, 1924.

The total number of ionisations per atom per second when exposed to black radiation on one side only is

$$\tfrac{1}{2}\int_{\nu_0}^{\infty}\psi(\nu)\frac{2h\nu^3}{c^2}e^{-h\nu/kT}d\nu.$$

This is approximately

$$\frac{64\pi^3}{3\sqrt{3}}\frac{Z^2e^6}{c^3h^4}kTe^{-h\nu_0/kT}.$$

For $\chi_0 = 11{\cdot}82$ volts, $T = 5740$, this comes to $Z^2 \times 1{\cdot}80 \times 10^{-3}$. If we apply this to the ionisation of the Ca^+ atom by adopting $Z = 2$, we get $7{\cdot}2 \times 10^{-3}$. This would mean that a Ca^+ atom is ionised to Ca^{++} every $1{\cdot}4 \times 10^2$ seconds. This must be a considerable overestimate of the frequency of ionisation.

We want now some criterion to see whether this rate of ionisation is compatible with a quasi-permanent high-level chromosphere. Every time a Ca^+ atom is ionised, a Ca^{++} atom will fall into the reversing layer, and as there is then a deficiency of Ca^+ atoms in the chromosphere a new Ca^+ atom will be ejected by radiation-pressure. For a chromosphere 10,000 km. thick, the average height from which the Ca^{++} atoms fall will be 5000 km. The time taken to fall will be, since $g = 2{\cdot}74 \times 10^4$,

$$\left(\frac{2 \times 5 \times 10^8}{2{\cdot}74 \times 10^4}\right)^{\frac{1}{2}} = 1{\cdot}9 \times 10^2 \text{ secs.}$$

This is of the same order of magnitude as the lower limit to the time taken to produce a Ca^{++} atom, so that these atoms are removed by gravity at least as fast as they are produced. As before mentioned, each transformation of a Ca^+ atom to a Ca^{++} atom will be followed by the ejection of another Ca^+ atom from the reversing layer. A quasi-permanent Ca^+ chromosphere is thus possible, even on this high estimate of ultra-violet solar radiation.

The point is, however, that a high-level chromosphere is a fact of observation. We know that Ca^+ atoms are present up to 10,000 km., and that radiation-pressure is adequate to support them. We see no mechanism by which Ca^{++} atoms can be supported to this height. Consequently we infer that the rate of photo-electric ionisation of Ca^+ atoms is too small to destroy the quasi-permanency of the chromosphere.

Summary.

1. Unsöld's pressure of $1{\cdot}7 \times 10^{-7}$ atmos. for the calcium chromosphere is an impossibility because the gas-pressure and radiation-pressure combined are totally inadequate to support a column at this pressure, 10,000 kms. high.

2. Unsöld's recent investigation of the equilibrium of a layer of Ca^+ and Ca^{++} atoms in thermodynamic equilibrium properly relates to the reversing layer and not to the chromosphere. His density distribution is a consequence of his overestimate of selective radiation-pressure, and seems physically unacceptable.

3. The equilibrium of calcium atoms on the sun is traced from the chromosphere to the reversing layer. The chromosphere consists almost entirely of Ca^+ atoms and free electrons.* The pressure is of the order of 10^{-13} atmos. At the base is a layer in local thermodynamic equilibrium, which we identify with the reversing layer. The ratio of Ca^+ atoms to Ca^{++} atoms at the boundary is about 0·02. Below the boundary the pressure increases, and with it the fractional concentration of Ca^+ atoms. The latter approaches its maximum value, almost unity, at a pressure of calcium atoms of 2×10^{-7} atmos. Below this again the pressure continues to increase, and the fractional concentration of Ca^+ atoms decreases and that of Ca atoms increases. The thickness down to the level of maximum Ca^+ is of the order of 100 kms. Some 30 kms. below this level the pressure of calcium atoms has increased to 5×10^{-5} atmos. The pressure of Ca^+ atoms in the chromosphere is continuous with that at the upper boundary of the reversing layer, the chromosphere consisting of Ca^+ atoms ejected from the supply available at the top of the reversing layer. In the chromosphere selective radiation-pressure just balances gravity, but as we descend below into the reversing layer it decreases as an exponential function of the optional thickness.

The above sketch of the structure of the calcium layers on the sun ignores for simplicity the effect of the presence of other elements, and it makes the assumption of a surface of demarcation between the chromosphere (in monochromatic radiative equilibrium) and the reversing layer (in local thermodynamic equilibrium). Actually the two must shade into one another. Further investigations are required to ascertain to what extent the results of the present paper require modification due to the transition of the one region into the other, both as regards the validity of the Saha formula and the variation of selective radiation-pressure.

Similar considerations will apply to other atoms. An important point to notice is that the pressure at the level of maximum concentration of an ionised atom (2×10^{-7} atmos. for Ca^+) depends on both first and second ionisation potentials.

4. Unsöld's theory of the formation of Fraunhofer lines remains fundamentally unaffected by the present investigation, because his calculated number of atoms are to be found lower down in the reversing layer, though at a higher pressure and compressed to a smaller thickness than he calculated.

5. In the Ca^+ chromosphere ionisation of an atom will occur from time to time. The resulting Ca^{++} atom will fall back into the reversing layer and be replaced by the ejection of another Ca^+ atom. A rough estimate is made of an upper limit to the frequency of photo-electric ionisation of Ca^+ atoms in the chromosphere.

* There are also Ca atoms supported by the line λ 4227, but it simplifies the description to ignore their presence in the chromosphere.

The University, Manchester :
1928 *January.*

Absorption Borders in the Spectrum of υ Sagittarii.

By Ralph H. Curtiss.

In a recent number of this journal * Dr. J. S. Plaskett calls attention to his detection in the spectrum of υ Sagittarii of three dark satellites accompanying hydrogen absorption lines on the side of shorter wave-length. Subsequently, the facts of this investigation have been published in greater detail.† When we have recognized the existence of emission about the hydrogen absorption, analogies of these lines with those of novæ and P Cygni stars are evident at once, as Dr. Plaskett points out. Emission, as such, is or has been observed clearly in υ Sagittarii at Hα,‡ usually with difficulty at Hβ,§ and it is probably present about Hγ, Hδ, and perhaps other Balmer lines of shorter wave-length, though not standing out from the continuous spectrum. This emission is variable.

It may facilitate the explanation of these hydrogen satellites of shorter wave-length, and it will clearly make their occurrence seem less unfamiliar, if they can be connected quantitatively with features observed in other bright line stars. It appears that this can be done by reference to observations of two of the oldest ‖ and best-known Class Be stars, β Lyræ and γ Cassiopeiæ. Nearly twenty years ago the writer ¶ detected and measured dark components (satellites) in the absorption borders of many hydrogen and helium emission lines in the spectrum of β Lyræ. At least four such components were found on each edge of the emission lines. Each of the components, whenever measured, was found displaced by the same amount from the normal centre of the line, when such displacements were expressed as velocities. It seemed probable that the intensities of the satellites were subject to change, as threefold increases in strength were shown in the published intensities of some of the lines at some phases as compared with others.

Again, in γ Cassiopeiæ the writer ** detected eight absorption components in the absorption borders of hydrogen emission lines—four in each border, as in β Lyræ, and strikingly enough in very nearly, if not quite, the same positions with reference to the centres of the emission lines. The lines were somewhat less measurable in the spectrum of γ Cassiopeiæ, and probably less intense but none the less suggestive of the character and arrangement that was found earlier in β Lyræ.

It is therefore of great interest to note that of Plaskett's component satellites in the spectrum of υ Sagittarii two appear to be identifiable in position with two of those in the spectra of β Lyræ and γ Cassiopeiæ, and the third appears to lie one-half of a space, so to speak, inside of the other components. This is made clear in the accompanying table.

* *M.N.*, **87**, 31, 1926.
† *Dom. Astroph. Obs.*, **4**, No. 4, 1927.
‡ *Astroph. Journ.*, **2**, 180, 1894.
§ *Annals H.C.O.*, **56**, 108; *see* **2**, 10.
‖ *Atti. d. Soc. Ital.*, **40**, Ser. 3, Tomo 1, Parte 1, p. 27, 1867.
¶ *Allegheny Publ.*, **2**, 109, 1911.
** *Publ. Univ. of Mich. Observatory*, **2**, 21, 1916.

Distances, expressed as Velocities from Central Lines, of Dark Hydrogen Satellites in Absorption Borders of Emission Lines.

No.	β Lyræ. km.	γ Cass. km.	υ Sag.	Mean. km.	Diff. km.
1	−471	−471	..	−471	
					92
2	−374	−383	..	−379	
					84
3	−282	−303	−299	−295	
					102
4	−186	−191	−202	−193	
					(40)
5	..	..	−153	−153	
Centre	±000	±000	±000		
6	+185	+191	..	+188	
					91
7	+262	+296	..	+279	
					93
8	+367	+377	..	+372	
					107
9	+498	+461	..	+479	
				Mean	95

In the table, components 3 and 4 are identical in position in all three stars within the limits of measurement. Satellite 5 was not measured in β Lyræ and γ Cassiopeiæ. If, however, we examine the spacing, as shown in the differences in the last column of the table, we find nearly uniform intervals averaging 95 kms. between adjacent lines in either border except between 4 and 5; and here the spacing is roughly half of the value we might expect. It seems probable that a part or a component of this line is covered by emission, and that the whole line, if visible, would lie in the expected position, 95 kms. inside of component 4. In β Lyræ the writer resolved component 8 into two lines. Possibly only one of two such lines was measurable in component 5 in υ Sagittarii. It is interesting to note also that the separation between components 4 and 6 is exactly four times the 95 km. interval between border components.

It would be interesting to know how widely the arrangement of components shown in the table is shared by other spectra in Class Be. It is interesting also to consider whether the components in the absorption borders should be regarded as dark lines or as bright ones, or both. The study of these features is technically difficult. Many spectra are necessary, and the frequency study used by the writer in γ Cassiopeiæ will be found useful.

A working picture of some of the complex lines in Be stars seems to be one in which a wide absorption band composed of many narrow lines has superposed on it a narrower emission line, which itself may be reversed. The emission may, in a given star, lie approximately in the centre of the wide absorption, leaving two equally strong absorption borders showing similar systems of satellites. In another case the emission may be permanently or periodically decentred in the absorption. In the P Cygni stars the emission is characteristically, though

possibly only periodically, decentred towards the red. As a result the violet absorption border is relatively strong and the red relatively faint or absent. The hydrogen lines in υ Sagittarii appear to belong to the latter class, with the violet edge of the underlying absorption found extensively exposed. Possibly some absorption components are visible in the red border of the emission of this star. Since the positions of such satellites are known in β Lyræ and γ Cassiopeiæ, they may be looked for in definite positions in υ Sagittarii.

It is by no means certain that the proposed working picture conforms to or suggests any theoretical explanation of these emission features. It serves as a basis on which the difficult absorption border details may be studied.

Ann Arbor, Michigan :
1927 *December* 24.

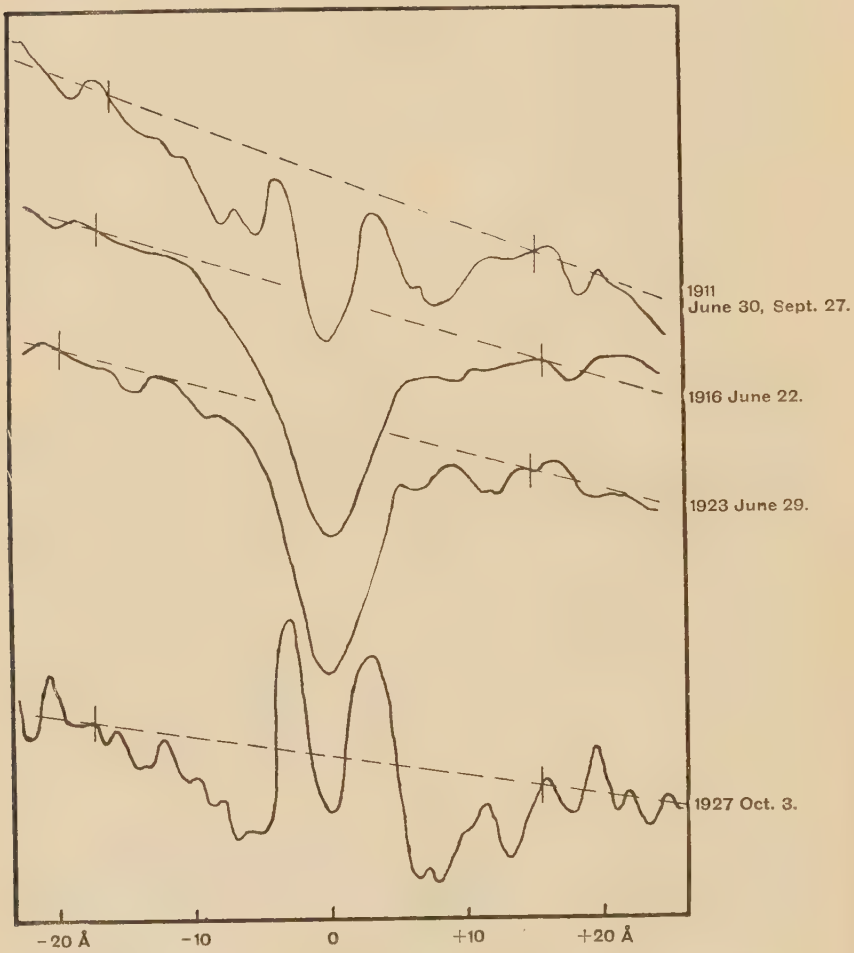
Renewed Visibility of Hydrogen Emission in the Spectrum of f^1 Cygni. By Ralph H. Curtiss.

The writer in previous papers * has called attention to the disappearance of hydrogen emission from the spectrum of f^1 *Cygni*. The record to date is based on the Harvard discovery observations, several two-prism Yerkes and one-prism Lick spectrograms of 1913 or earlier, and a series of sixty-five spectrograms made at Ann Arbor in the years 1911 to 1927. This record follows :—

1904. The Balmer series, Hβ to Hζ, bright. Hβ, strong emission.
1909. Two-prism dispersion shows no emission visible in photographic region.
1911. Emission visible at Hβ and Hγ on contrasty plates under single-prism dispersion.
1912. Emission indistinctly visible at Hα and Hβ.
1913. Two-prism dispersion shows no emission in the photographic region, except possibly a trace at Hβ.
1915. A trace of narrow emission visible at Hβ on edge of shorter wave-length, under one-prism dispersion.
1916. No emission visible even at Hα.
1917–1920. No emission visible in photographic region.
1921. Indistinct emission observed at Hβ and suspected at Hγ.
1923. Weak emission at Hβ visible on edge of shorter wave-length.
1924. Weak emission at Hβ well shown on edge of shorter wave-length ; probably at Hγ.
1925. Emission still weak, but visible probably on both edges of the central absorption of Hβ.
1926. Emission well defined at Hβ and visible at Hγ. It is stronger on the edge of greater wave-length.
1927. Emission clearly shown at Hβ with typical structure, including central absorption, double emission, and emission borders.

* *Publications of the University of Michigan Observatory*, **2**, 36; **3**, 20.

Apparently hydrogen emission was strong in f^1 Cygni in 1904, but faded, rapidly at first, and reached a minimum in 1918 or thereabouts. At present it is gaining strength. Small secondary variations were probably present also.



Intensity Curves of Hβ in the Spectrum of f^1 Cygni. (Densities on the negative increase upward.)

As the emission disappeared at Hβ its last manifestation took the form of a narrow line on the edge of shorter wave-length. On its return it took this same form. As the emission waned, the central absorption appeared to strengthen and widen, taking the place of the emission, at least partially, and narrowing again as the emission returned.

The accompanying curves were made with a thermo-electric self-registering photometer and enlarged five-fold vertically. The first curve is a mean of two records. The slope in each case, as indicated by the discontinuous line, is due almost entirely to the variation in sensibility of the ordinary plate with wave-length. The steeper slopes are characteristic of thinner negatives. The double emission is clearly shown in 1911, and again in 1927. The existence in 1911, and again in 1927, of several components in the absorption borders of Hβ is well shown. Short vertical lines on the curves indicate estimated limits of the wide absorption. These curves are, however, not competent to establish any variation in such limits, which probably do not undergo any marked changes.

The photometric curves bring out (not too clearly) the fact that the emission of H-Beta in 1927 was narrower and the two components of this emission closer together than in 1911, when this feature was fading from visibility. The width of the H-Beta emission in 1911 was 10·51 angströms, whereas in 1927 this width was 8·11 angströms.

Obviously there is no assurance that the spectrum variations of f^1 Cygni are periodic. If they are cyclic, the period is evidently very long—possibly twenty-five or thirty years in duration. The writer has shown that the radial velocity of this star varies also.* This supports the guess that the spectrum variations are periodic, but is by no means conclusive. It may require many years to settle the matter.

This star may be removed now from the list of objects like Pleione, which have lost their hydrogen emission. Indeed, its behaviour raises the question whether any star whose emission has faded out within the relatively short interval covered by stellar spectroscopic research will not quite surely possess it again. Relatively great intrinsic brightness with high temperature radiation and an extensive atmosphere of low density are conditions favourable to the presence of visible emission in the light of class B stars. Of these conditions, brightness and temperature are not known to be variable in these stars. The atmospheres may be variable, whether as the result of " tides " produced by a companion, or as the effect of motion through space occupied by rarefied matter, or as a consequence of pulsatory waves of alternately increased and decreased density moving upward through the atmosphere. If the visible emission in class B spectra is due to fluorescence † or a similar phenomenon, slight changes in atmospheric density or in the quality of the exciting emission, coming probably from the photosphere of the star, may be competent to produce many of the observed variations in the hydrogen bright lines.

* *Publications of the University of Michigan Observatory*, **2**, 36; **3**, 20.
† Rosseland, *Mt. Wilson Contr.*, No. 309.

Ann Arbor, Michigan :
1928 *January* 11.

On the Time of Relaxation of Closed Stellar Systems.

By S. Rosseland.

(*Communicated by Prof. A. S. Eddington.*)

1. The problem of calculating the time of relaxation of a stellar system has been attacked by several authors. The most rational procedure is probably that of Charlier,* which is based directly upon his general form of statistical mechanics. This method leads, however, into mathematical prolixities which tend to mask the simple features of the phenomena in question. Jeans † and Eddington,‡ on the other hand, proceeded in a more direct way, by considering the effect of chance encounters on the direction of the velocity of a given star. The simple procedure used by Eddington and Jeans involves some unattractive features due to the fact that it becomes necessary to disregard all passages taking place beyond a certain distance. This difficulty is by no means inherent in the method, as it is solely due to the simplifying assumption that a calculation of the linear sum of the chance deviations will suffice to form an estimate of the effect of encounters. However, probability theory requires the composition of the squares of the contributions from independent encounters, and when this is taken care of, the difficulty in question vanishes. How this works out will be shown in the following where we, moreover, shall consider directly the rate at which the probability distribution of the kinetic energy of the stars approaches equipartition, this being the point of view which is most in keeping with the methods of ordinary statistical mechanics.

2. Consider an individual star during its motion through a stellar system in dynamical equilibrium. Let Q be the kinetic energy gained by this star in a particular passage with another star. The quantity Q will depend upon the masses and initial velocities of the two stars as well as upon their least distance of approach. Assume that we observe the star through an interval of time sufficiently long for it to have suffered a very large number of passages, involving a continuous range of Q-values. On an average, successive Q-values must be independent of one another to a high degree of approximation such as to combine according to a normal Gaussian error law. The probability that a particular Q shall fall in the range from Q to $Q + dQ$ is therefore

$$W(Q)dQ = \frac{1}{\sqrt{2\pi}L} e^{-\frac{(Q-E)^2}{2L^2}} dQ \quad . \quad . \quad . \quad (1)$$

where

$$E = \sum_i Q_i \quad . \quad . \quad . \quad . \quad (2)$$

* C. V. L. Charlier, *Lund Meddelanden*, **28**, Nr. 16, 1917.
† *Cf.* "Problems in Cosmogony, etc."
‡ *Cf.* "Stellar Motion, etc."

is the arithmetic mean increase in kinetic energy of the star in the given time-interval and due to encounters, while

$$L^2 = \sum_i Q_i^2 \quad . \quad . \quad . \quad . \quad (3)$$

is the quadratic mean increase of kinetic energy in encounters in the same interval of time.

As a function of the time the quantities E and L will behave very differently. Thus E will rarely exceed the average kinetic energy of a star belonging to the system. If the particular star in question has an initial kinetic energy much in excess of the average, E may increase until it becomes of the order of this energy, but will rarely increase any further, and from then on it will most frequently oscillate in the interval $\pm$ T, where T is the average kinetic energy of a star in the system. The quantity L^2, on the other hand, will increase indefinitely, since each encounter gives a positive contribution, and apart from minor oscillations it will increase proportionately to the time. In order to see what this means we follow the motion of an individual star in the stellar universe. At the time $t = 0$ we measure its coordinates and velocity components, so that, knowing the average mass distribution in the system, we could calculate a mean orbit for the star and predict its kinetic energy in this orbit for any future time. Owing to encounters the star strays more and more from its predicted path and at the same time its kinetic energy differs from the value calculated for the mean orbit. Whether the star strays to one side or the other, and whether its kinetic energy is above or below the mean value, depend upon circumstances which are uncontrollable individually, but can be mastered to some extent by probability considerations. As long as the dispersion in kinetic energy L is small compared to the kinetic energy of the star itself, the encounters have not had a material influence on the motion of the star as far as kinetic energy is concerned. But inasmuch as L grows approximately proportionally to the square root of the time, the deviation grows worse and worse, and finally a state is reached in which it must be admitted that the calculation of a mean orbit is no longer of any value for an identification of the star in question. The fixation of this time is largely a matter of choice. We shall here adopt the time necessary for L to become of the same order as the initial energy of the star if this is in excess of the average; otherwise it seems reasonable to adopt the time which makes L equal to the average kinetic energy of a star of the system. In either case we shall call the time-interval in question the *time of relaxation of the star.*

3. In order to calculate L for a given star it is necessary first to know the expression for the individual loss of energy in an encounter. Let $\mathbf{V}'$ and $\mathbf{V}''$ be the vectorial velocities of the star before and after the encounter, and denote its mass by m. The loss of energy in the encounter is then

$$Q = \frac{m}{2}(\mathbf{V}'^2 - \mathbf{V}''^2) \quad . \quad . \quad . \quad . \quad (4)$$

Let $\mathbf{V_g}$ be the constant velocity of the centre of mass of the two stars, and $\mathbf{v}$ the velocity of the passing star relative to this centre. Relative to a co-ordinate system moving with the velocity $\mathbf{V_g}$ the orbit of the star is a hyperbola, and the absolute value of $\mathbf{v}$ will be the same for any two points of the orbit which are symmetrically situated with respect to the centre of gravity. This gives directly

$$Q = \frac{m}{2}\{(\mathbf{V_g} + \mathbf{v}')^2 - (\mathbf{V_g} + \mathbf{v}'')^2\} = mV_g v(\cos\psi_1 - \cos\psi_2) \qquad (5)$$

where V_g and v are the absolute values of $\mathbf{V_g}$ and $\mathbf{v}'$, $\mathbf{v}''$ respectively, ψ_1 is the angle between $\mathbf{V_g}$ and $\mathbf{v}'$ and ψ_2 the angle between $\mathbf{V_g}$ and $\mathbf{v}''$.

Consider the simple case when the initial velocity of the star under consideration is largely in excess of the average velocity of the stars of the system, so that these stars may be considered as being at rest at the beginning of an encounter. We have then

$$\cos\psi_1 = 1, \qquad V_g = \frac{mV}{(m+M)}, \quad \text{and} \quad v = \frac{M}{m+M}\cdot V \qquad (6)$$

where M is the mass of the star which is initially at rest, and V the absolute value of $\mathbf{V}'$. Introducing these values in (5) we find, dropping the index to ψ_2

$$Q = \frac{2m^2M}{(m+M)^2}V^2\sin^2\frac{\psi}{2} \qquad . \quad . \quad . \qquad (7)$$

This expression is valid for any kind of conservative fields of force. Adopting the Newtonian law we have the well-known result that

$$\text{Cotang}\frac{\psi}{2} = \frac{pV^2}{G(M+m)}, \quad \text{giving} \quad \sin^2\frac{\psi}{2} = \frac{1}{1 + \frac{V^4}{G^2(M+m)^2}\cdot p^2} \qquad (8)$$

where p is the distance from the star initially at rest to the initial direction of motion of the passing star, and G the constant of gravitation. The average number of passages in the time-interval dt, with respect to stars with mass M, and for which p is in the interval between p and $p + dp$, is

$$d\sigma = 2\pi N p dp V dt \qquad . \quad . \quad . \quad . \qquad (9)$$

This gives a contribution to L^2 of

$$dL^2 = \int_p Q^2 d\sigma = \pi V dt N \left\{\frac{2m^2M}{(m+M)^2}\right\}^2 \cdot V^4 \int_0^\infty \frac{2pdp}{\left(1 + \frac{p^2V^4}{G^2(M+m)^2}\right)^2} \qquad (10a)$$

or performing the integration

$$dL^2 = 4\pi N G^2 m^4 (1 + m/M)^{-2} \cdot V dt \qquad . \quad . \qquad (10b)$$

Let a bar denote a time-average following the motion of the star.

The quadratic mean loss of kinetic energy in the time-interval t then assumes the form

$$L^2 = t4\pi G^2 m^4 \sum_i \overline{N_i V}(1 + m/M_i)^{-2} \quad . \quad . \quad (11)$$

where we have performed a summation over stars of different masses. Let V_0 be the velocity of the star at the time $t = 0$. We have then agreed that the time of relaxation τ will be given by (11) by putting $L = \frac{1}{2}mV_0^2$, that is

$$\tau^{-1} = 16\pi G^2 \frac{m^2}{V_0^4} \sum_i \overline{VN}_i(1 + m/M_i)^{-2} \quad . \quad . \quad (12)$$

In order to estimate the time of relaxation of our local star-system by this formula we may assume all stars to have the same average mass $m = 4 \cdot 10^{33} g$ (twice the mass of the sun), an average velocity of say 20 km. sec.$^{-1}$, and a number of stars per unit volume of one star per 10 cubic parsec, which corresponds to the conditions near the sun. We then find

$$\tau = 10^{14} \text{ years.}$$

4. It may be of interest also to consider the case when the initial velocities of the stars, other than the one whose motion we are studying, are not neglected. The following considerations will give an indication of how the motion affects the dispersion in kinetic energy. Consider the stars in a certain volume element $d\sigma$, which lies outside the path of the moving star under consideration. Let M be the mass and $\mathbf{V}$ the initial velocity of a star in $d\sigma$, and let $d\mathbf{V}$ be the vectorial increment to $\mathbf{V}$ due to the action of the passing star. We have then for this particular passage

$$Q = \tfrac{1}{2}M\{(\mathbf{V} + d\mathbf{V})^2 - \mathbf{V}^2\} = \tfrac{1}{2}Md\mathrm{V}^2 + MVd\mathrm{V}\cos\alpha \quad . \quad (13)$$

where α is the angle between $\mathbf{V}$ and $d\mathbf{V}$. The first term on the right-hand side of this expression is independent of V and is therefore identical with the Q-value used in the preceding section, and which we here shall denote by Q_0. We find therefore by averaging over all stars in $d\sigma$

$$\overline{Q^2} = \overline{\{\tfrac{1}{2}Md\mathrm{V}^2 + MVd\mathrm{V}\cos\alpha\}^2} = Q_0^2 + Q_0 2M\overline{V^2\cos^2\alpha} + 2Q_0 M\overline{Vd\mathrm{V}\cos\alpha} \quad (14)$$

If V, dV, and α are uncorrelated we have

$$\overline{V^2\cos^2\alpha} = \tfrac{1}{3}\overline{V^2}, \quad \text{and} \quad \overline{V \,.\, d\mathrm{V} \,.\, \cos\alpha} = 0 \quad . \quad . \quad (15)$$

giving finally

$$\overline{Q^2} = Q_0^2 + \tfrac{2}{3}M\overline{V^2} \,.\, Q_0 \quad . \quad . \quad . \quad (16)$$

showing that the motion of the stars of the system will tend to increase the dispersion in kinetic energy, and hence to decrease the time of relaxation. Formula (16) is not suited for a quantitative estimate of this time, since there will be an appreciable correlation between dV and α in the vicinity of the path of the passing star.

5. If we should try to calculate the mean loss of energy of a star in a time-interval dt, on using expression (7), for the loss in an individual encounter we should find an excessive value, which becomes logarithmically infinite for a system extending to infinity. This is a peculiarity of the inverse square law which is well known from kinetic gas theory. The infinity in question is not real, however, and is solely due to the fact that (7) can only give an upper limit to the loss of energy in an encounter. This makes no difference in a calculation of the energy dispersion, which only depends upon relatively close encounters. On the other hand, it makes all the difference in case of a calculation of the linear loss of energy, which is essentially determined by the combined gravitational attraction of the stars, supposed to form a continuous mass distribution.

The error in question arises from the fact that formula (7) supposes the stars initially to be at an infinite distance from each other, and to separate to an infinite distance after the encounter. Only so is ψ equal to the angle between the asymptotes of the relative orbit, and given by (8). When a star only moves a small distance $dx = Vdt$ it is clear that the two stars only have completed a part of their relative orbit, so that the true value of ψ is smaller than the one given by (8). For close passages the difference matters little; but for more distant passages the magnitude of dt must be taken into consideration. The peculiarity of the inverse square law in this connection consists therefore in the breakdown of the fundamental assumption of gas theory, that passages involving an appreciable change of energy always take place in a time-interval which is short in comparison to macroscopic time-intervals.

This fact shows, as is well known, that a statistical stellar mechanics will have to neglect encounters taking place at great distances. The fixation of the upper limit to the distance from the path of a star, within which the exchange of energy and momentum is considered to take place by individual passages, requires certain precautions, which do not seem to have been properly attended to by previous investigators. It follows from (10a) that the main contribution to the dispersion in kinetic energy comes from passages for which p is less than $\alpha G(M+m)/V^2$, where α is a numerical multiplier of order unity. This limit is therefore essentially a function of the relative velocity of the stars. On the other hand, the consideration of the frequency of multiple passages would seem to set an upper limit which is independent of the velocities, and of the order of the mean distance between adjacent stars. Charlier (*loc. cit.*), for instance, based his statistical mechanics on the assumption that this limit plays the principal part. A numerical estimate indicates that this is permissible for the galactic system; but whether or not it will work for a system selected at random will depend upon individual circumstances.

Institute of Theoretical Physics, Copenhagen:
1927 *November.*

On Some Relations between the Masses, Luminosities, and Spectra among Eclipsing Binaries. By G. Shajn.

Recently Eddington called attention to the fact that the observed relations between spectra and luminosities of the constituents in double stars and globular clusters are in conflict with theoretical considerations : * the more luminous (more massive) components are in the earliest diffuse stage, and this appears to indicate that the smaller mass evolves more rapidly than the greater. The theory seems to be against it. The object of the present note is to point out some relations between the masses, spectra, and densities of the components in eclipsing binaries which may be of interest for the problem in question.

The light-curve furnishes amongst other data the ratio J_1/J_2 of the surface brightness of the components. Hence, if the spectral type or effective temperature is observed, the effective temperature of the secondary can be found with tolerable accuracy from the formula

$$J_1/J_2 = \frac{e^{\frac{c}{\lambda T_2}} - 1}{e^{\frac{c}{\lambda T_1}} - 1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The light-curve gives also the luminosities of the components in terms of brightness of the system, and this will suggest an approximation to the mass-ratio on the basis of the theory of radiative equilibrium. The photometric results are from Shapley's discussion,† and some more recent papers. The " darkened " solution is preferred, except for a few stars for which only the " uniform " solution is known. Considerable errors of the elements of eclipsing binaries are inevitable, and the results are affected by several uncertainties, but it seems improbable that more complete data will radically modify them.

The mass-ratios in double stars indicate that the brighter component is usually associated with larger mass. There are known exceptions, but in most cases we cannot trust the mass-ratio deduced from discussion of the absolute orbits.

The spectroscopic binaries give the same result, and in this case the criterion for the brightness of the components is the intensity of spectral lines. These estimates are in most cases uncertain. But in spectroscopic binaries there are also exceptions, for instance β Lyræ. A real discovery of such systems would be very important from the theoretical point of view. In this relation the eclipsing binaries are of considerable interest on account of the fact that the brighter component is usually associated with smaller radius and larger density.

The mass-ratios are computed in the way indicated in *M.N.*, **85**, 245. The method is essentially the same as that employed by Eddington ‡ (mass-ratio for β Persei). In Table I I give a list of 15 eclipsing binaries which have known orbits (spectroscopic) for both components. It may

* Eddington, *The Internal Constitution of the Stars*, p. 311.

† Shapley, " A Study of the Orbits of Eclipsing Binaries," *Contr. Princ. Univ. Observ.*, No. 3.

‡ *I.C.S.*, p. 209.

be used to check the method employed in this note. M_2/M_1 (observed) are taken from Moore's Catalogue of spectroscopic orbits. The last column gives the grade of the photometric orbit as classified by Shapley, grade 1 being the most trustworthy.

TABLE I.

Star.	Δm.	M_2/M_1 (calc.).	M_2/M_1 (observ.).	O−C.	Class.	Star.	Δm.	M_2/M_1 (calc.).	M_2/M_1 (observ.).	O−C.	Class.
TX Herc.	·00	1·00	·86	−·14	3	*u* Herc.	1·00	·55	·39	−·16	1
TV Cass.	1·83	·62	·58	−·04	1	U Ophiu.	·15	·93	·88	−·07	1
Z Vulp.	1·19	·51	·45	−·06	2	W Urs. M.	1·25	·75	·71	−·04	1
H.D. 1337	·90	·64	·93	+·29		Y Cygni	·35	·84	·92	+·08	3
Z Herc.	·31	1·03	·87	−·16	2	Castor C	·26	·94	·90	−·04	
U Coron.	1·72	·57	·39	−·18	2	S Antil.	·77	·83	·56	−·27	
RX Herc.	·00	1·00	·96	−·04	1	β Pers.	2·83	·40	·20	−·20	1
β Aurig.	1·00	1·00	·98	−·02	1						

Notes.

1. β Lyræ and RS Vulpec. are excluded from discussion on account of the peculiar spectrum of β Lyræ and the uncertain photometric orbit of RS Vulpeculæ.
2. For H.D. 1337 and Y Cygni the effective temperature of both components is assumed to be equal, 26,000°.
3. The spectroscopic orbit of RX Herc. is doubtful.
4. For W Ursæ Maj. the elements are for the " uniform " solution.
5. For Castor C and S Antilæ the photometric orbits are uncertain.
6. For β Persei the ratio M_2/M_1 is probably affected by distant companion.

Pearce has found * that Eddington's mass-luminosity relation does not satisfactorily predict the absolute magnitude of H.D. 1337.

It must be noted that the residuals depend to a large extent on the temperature scale. For the luminosities Eddington gives †

$$\delta(O - C) = -8\delta(\log_{10}T_e) \quad . \quad . \quad . \quad (2)$$

We have adopted the temperature scale based on Saha's theory of stellar spectra. ‡ In spite of inevitable errors in the observational data the agreement is fair. From Table I we have

TABLE I'.

Limits Δm.	Mean Δm (vis.).	Mean $\Delta m'$ (bolom.).	M_2/M_1 (calc.).	M_2/M_1 (observ.).	O−C.	*n*.
0·00–0·15	0·04	0·04	0·98	·92	−·06	4
0·31–0·45	0·35	0·23	0·94	·90	−·04	3
0·77–1·25	1·02	1·45	0·66	·61	−·05	5
1·72–2·83	2·13	2·73	0·53	·39	−·14	3

The residuals are mostly negative. It will be shown later that the companions exhibit such peculiarities that it is difficult to consider the

* *Publ. Dom. Astroph. Observ.*, **3**, No. 13, p. 303.
† *I.C.S.*, p. 156. ‡ Payne, *Stellar Atmospheres*, p. 33.

theory to be responsible for these negative residuals. It is possible, however, that some other form of mass-luminosity relation may give a more satisfactory agreement for these stars.* In spite of uncertainties mentioned above Tables I and I′ test the reliability of the method employed in this note. This may be especially justified in a statistical study. The influence of the negative residuals will be noted below.

Table I′ definitely favours the supposition that not only the brighter component is associated with larger mass, but that with increasing difference in brightness the difference in masses increases. The role of the temperature in the mass-luminosity relation is accounted for by the term $2 \log \frac{T_e}{5200}$. It is possible that the influence of the temperature is more considerable. In any case we give also the bolometric difference $\Delta m'$ as the mass-ratio is in close connection with it.

2. In this discussion are included 108 eclipsing variables, the period of which is less than 40 days. The results are given in Table II, which is self-explanatory.

Table II.

Spectrum.	Δm (visual).	m.e.	$\Delta m'$ (bolom.)	M_1.	M_2.	M_2/M_1.	n.
B0–B5	1·55	±·30	2·96	9·27	4·00	·43	13
B8–B9	1·48	·39	2·03	3·57	2·27	·64	10
A0–A2	1·93	·36	2·02	2·43	1·63	·67	57
A3–A5	1·55	·36	1·45	1·90	1·47	·77	9
F	1·04	·25	·86	1·36	1·21	·89	15
G	·74	·25	·40	1·19	1·12	·94	3
M	·26	..	·33	0·60	0·57	·94	1

The mean error $\sqrt{\frac{\Delta^2}{n(n-1)}}$ refers to Δm. It is evident that the dispersion in Δm is large. Notwithstanding this the increase of mass-ratio with advancing type B0–B5, B8–B9 . . . is seen in a more regular manner. The average mass-ratio tends to approach unity for the systems considered to be furthest advanced. This conclusion may be connected with the loss of mass by radiation. We have here an analogy between eclipsing variables, double stars,† and spectroscopic binaries.‡ The eclipsing variables exhibit this phenomenon in a very evident manner. The uniformity appears to indicate evolutionary progress. However, allowance must be made for selection effect.§ It was mentioned above that the residuals are negative for the computed mass-ratios, and it seems that they increase with increasing Δm. As Δm decreases with advancing type it follows that if allowance for the residuals be made, the dependence of mass-ratio on the spectrum will be still more evident. But the residuals may also depend on the spectrum, and we cannot account for this.

* Jeans, *M.N.*, **85**, 208.
† Vogt, *Zeitschr. f. Physik*, **26**, 139; Shajn, *M.N.*, **85**, 245.
‡ Aitken, *The Binary Stars*, 207; Vogt, *A.N.*, **225**, 315.
§ *M.N.*, **85**, 249.

3. The ratio of the relative intensity per unit area of the components of eclipsing binaries is equal to

$$J_1/J_2 = \frac{L_1 r_2^2}{L_2 r_1^2}.$$

It is convenient to express this ratio as the surface brightness of the brighter star in terms of the fainter one. Table III exhibits the relation between Δm and J_1/J_2 for eclipsing binaries of all spectral classes together.

TABLE III.

(All types together.)

Limits of Δm.	J_1/J_2.	m.e.	n.
·00– ·50	2·3	± ·6	24
·51–1·00	4·9	1·4	10
1·01–1·50	12·2	2·8	16
1·51–2·00	9·7	1·7	16
2·01–2·50	19·3 (14·6) *	5·3 (2·9)	17 (16)
2·51–3·00	19·3	2·6	15
3·01–3·50	22·1	5·3	6
3·51–4·00	51·9	9·8	4

As the surface brightness is a function of the spectrum, the relation represented in Table III leaves no doubt that with increasing magnitude-difference of the components the difference of their spectral characteristics increases, otherwise, the greater the difference of the masses of the components the more different their surface conditions.

In Table IV we give the results for the homogeneous group B8–A5 which appears to indicate the same peculiarity as all types together.

TABLE IV.

(B8–A5.)

Limits of Δm.	J_1/J_2.	m.e.	T_e (secondary) computed.	Spectrum (secondary) estimated.	n.
·00– ·50	1·2	± ·1	9400	A1–A2	12
·51–1·00	6·7	1·7	5900	F8	4
1·01–1·50	12·5	3·2	5200	G3	12
1·51–2·00	12·0	1·8	5200	G3	12
2·01–2·50	19·2 (13·8) *	6·4 (2·9)	4800	G7	16
2·51–3·00	20·1	2·8	4700	G7	13
3·01–4·00	33·6	5·1	4300	G8	7

The effective temperatures of the secondary stars were computed from Planck's formula and checked by Table XVI derived by Eddington.†

* Excluding TT Hydræ with exceptional $J_1/J_2 = 95$.

† Eddington, *I.C.S.*, p. 139.

It is of interest to note that the relation mentioned in Tables III and IV is repeated in double stars (*M.N.*, **85**, 278). Instead of Δm there is given the mass-ratio, but, as is shown above, there is a close connection between them. However, the difference of spectral characteristics of the components in eclipsing binaries is much larger than that in double stars, the magnitude difference being the same.

The results of comparison of J_1/J_2 with the spectrum of the primary are given in Table V.

TABLE V.

Spectrum.	T_e (primary).	J_1/J_2.	T_e (second.) computed.	Spectrum (second.) estimated.	n.
B0–B5	17000	11·7	6800	F5	13
B8–B9	12500	10·8	6000	F8	10
A0–A2	9500	16·4	4800	G7	57
A3–A5	8700	10·2	4900	G5	9
F	7200	6·3	4700	G7	15
G	5300	5·2	4000	K1	3
M	3000	1·1	3000	M	1

It is evident that with advancing type (except B class) J_1/J_2 on the average decreases; besides, the effective temperature of the secondary also decreases. Recently it was shown that double stars exhibit the same peculiarity, but this analogy is only qualitative.* It is obvious that Table V is a consequence of Tables II and IV.

4. If we assume that the components of an eclipsing binary are equally massive we may derive the values of their densities in terms of the sun as soon as we know the period of revolution in days and the dimensions of the stars relative to their distance apart (equal-mass density). Shapley, using an empirical formula † $M_b = 0{\cdot}4 + 1{\cdot}2L_b$ gives the density of each of the components, where M_b is expressed in terms of one-half the total mass of the system. It appears to me that a better approximation to the truth may be reached if we use the mean mass-ratios taken from Table II. The densities of the components ρ_1 and ρ_2 are connected with the equal-mass densities $\bar{\rho}_1$ and $\bar{\rho}_2$ by the following formula:

$$\rho_1 = \frac{2\bar{\rho}_1}{1+\alpha} \quad \text{and} \quad \rho_2 = \frac{\bar{\rho}_2}{2}(1+\alpha) \qquad . \qquad . \qquad (3)$$

where α is the mass-ratio. The results of computations are given in Table VI. It is evident that the more massive components belong to the main series. The increase of the density with advancing type is brought out in the most evident manner, especially for the primary. The number of late-type stars is very small, nevertheless it appears that this relation is real. Attention should be called to the ratio ρ_2/ρ_1 which appears to increase with advancing type, except for B stars. This peculiarity may be compared with the increase of mass-ratio with advancing type (Table II). It seems to be remarkable that in eclipsing

* *M.N.*, **85**, 250.

† Shapley, *Contr. Princ. Univ. Observ.*, No. 3, p. 123.

binaries the spectrum and density of the components are in such close connection with the difference of their masses.

TABLE VI.

Spectrum.	ρ_1.	m.e.	ρ_2.	m.e.	ρ_2/ρ_1.	n.
Bo–B5	·074	±·024	·069	±·024	·93	13
B8–B9	·132	·024	·095	·028	·72	10
Ao–A2	·163	·020	·058	·009	·36	57
A3–A5	·256	·087	·159	·084	·61	9
F	·340	·063	·237	·053	·70	15
G	·740	·558	·621	·609	·84	3
M	1·400	..	1·800	..	1·29	1

5. In order to understand the facts mentioned above it is important to know the absolute magnitudes of eclipsing binaries. In the paper by Russell and Shapley * are given the hypothetical parallaxes of 90 eclipsing binaries. The method of deriving the parallaxes cannot be considered reliable. Assuming the mass and surface brightness to be known for each spectral class in terms of the sun, and using the value of the radius of the brighter star computed on the assumption that the mass of the system is twice that of the sun, the absolute magnitudes of the components may be found. The absolute magnitudes for 90 systems were derived from Shapley's paper and computed for 18 other eclipsing binaries. For the sake of uniformity it seems advisable to adopt the mass and surface brightness derived by Russell, although more recent data exist. It is obvious that at last the computed absolute magnitudes depend on the adopted value of mass and surface brightness. The results are given in Table VII. In the last column are given for comparison the absolute magnitudes of Adams and Joy for early-type stars (the average between n and s was assumed).† The agreement between the second and fifth columns is not so satisfactory, and there is a systematic difference of about 0·6. Using the values of Δm taken from Table II and the absolute magnitudes of the primaries (average between second and fifth columns) we are able to derive the mean absolute magnitudes of the secondary (third column).

TABLE VII.

Spectrum.	Absolute Magnitude.		n.	Absolute Magnitude (Adams and Joy).
	Primary.	Secondary.		Primary.
Bo–B5	−1·8	−0·3	13	From −3·1 to −0·8
B8–B9	−0·1	+1·5	10	0·0
Ao–A2	+0·1	+2·4	57	+0·9
A3–A5	+1·0	+3·0	9	+1·8
F	+2·6	+3·9	15	+3·1 (Fo–F2)
G	+3·7	+4·5	3	

* *Aph. J.*, 1913, **38**, 158; *Contrib. Princ. Univ. Observ.*, No. 3.
† *Aph. J.*, 1922, **56**, 242.

On the basis of Table VII it is possible to estimate very roughly the spectral class of the secondary which appears to be F in eclipsing binaries of type A, and F–G and K in those of type F and G. But it is difficult to estimate the spectrum of the secondary in the systems of B type. It is known that in double stars of type B the primary is usually associated with a secondary of early type, and that there is a large dispersion in mass and absolute magnitude, within the limits of the B-type systems.* It is important to note that the estimated spectrum of the secondary on the basis of absolute magnitude leads to a somewhat earlier type than that which follows from J_1/J_2. Another surprising fact is that the companions of the systems A, F, and G may be interpreted as belonging to the main series, and this is difficult to reconcile with their low density. However, it is to be remembered that the computed absolute magnitudes are not independent of the adopted values of masses. I think that for B and A systems the absolute magnitudes are more trustworthy. Among 18 eclipsing binaries of type F and G there are known two trigonometric or spectroscopic parallaxes (R Can. Major of type F + 1·7 and W Urs. Major of type G + 4·0), and they are in agreement with the hypothetical values.

6. We shall now add some further evidence in favour of the peculiarity found above, namely, that the spectrum of the secondary is of a later type than that of the primary. It is obvious that selection is an important factor in such a question. The distribution of the eclipsing binaries according to Δm is given in Table III. Three combinations of the spectra of the components are possible: (1) the spectra are identical, (2) the brighter component is of a later type, and (3) the brighter component is of an earlier type than the fainter one. Among double stars and spectroscopic binaries there is a considerable percentage of systems with small Δm, and they are usually associated with very small spectrum difference between the components. An analogy should be expected for eclipsing binaries, and the first line of Table III supports this. In the second case the photographic difference Δm is smaller than the visual, and the spectrum of the system must be composite since it is sensibly affected by that of the secondary. It is known that among eclipsing binaries the composite spectra are extremely rare. On the contrary, among close double stars (giant systems of type F–A, G–A, and K–A) composite spectra are very frequent.† In the third case the photographic difference Δm is greater than the visual, and the spectrum of the system during the constant phase will be affected by that of the secondary but very little, the other circumstances being the same. Perhaps this is responsible for the large percentage (about 13) of peculiar spectra among the eclipsing binaries. The consideration of the general character of the spectrum of eclipsing binaries seems to support the fact that the spectrum of the secondary is of a later type than that of the primary.

During the constant minimum phase of a total primary eclipse the light is wholly that of the faint companion, but because of the faintness of the star and short duration, spectroscopic observations are possible

* *M.N.*, **85**, 248.
† *A.N.*, **228**, 337.

in a few cases only. We can, however, estimate the colour-indices of the faint companions by determining photographically and visually the range of variation at the primary minimum. In order to obtain reliable results the eclipse must be total or nearly total, so that the light of the brighter star does not predominate over the light of the faint companion. There are 16 eclipsing binaries for which either the ratio of the amplitudes photogr./photom. or the spectra of both components are known with more or less accuracy. These data are mostly due to Russell, Shapley, and Seares.* For the sake of brevity I give only the mean value 1·38 of ratio of the amplitudes photogr./photom. Whenever the relative colour-indices of the components have been determined from the difference between photographic and visual ranges at total eclipse the faint star has always been found to be the redder. This is direct evidence in favour of an analogy between eclipsing binaries and dwarf double stars with regard to the spectral relation between the components.

7. All the considerations mentioned above (absolute magnitude, density, spectrum relation, and analogies with double stars) leave no doubt that the primaries of the eclipsing binaries belong to the main series. With regard to the secondary the data are contradictory. The secondaries may be interpreted as stars of the main series according to luminosity and as giants according to the density, excluding perhaps the systems of late types. On the other side, there is a contradiction between effective temperature and absolute magnitude: the companions are too bright for their computed effective temperature if they belong to the main series, and too faint if they are giants. Moreover, the companions are a little denser than the normal giants. It is very difficult to reconcile all these facts. The position of the companions of eclipsing binaries in the Russell-Hertzsprung diagram is of interest, especially if this diagram be interpreted as indicating the course of evolution of an individual star.

It appears to me that the companions are peculiar stars, and this suggestion is based on the following facts: (1) The position of the companions on the luminosity-spectrum diagram of Russell and Hertzsprung is unusual as compared with single stars (between giants and dwarfs); (2) the density does not correspond to the effective temperature; (3) there is no correspondence between the luminosity and effective temperature; (4) the difference of the surface brightness of the components is much larger in eclipsing binaries than in double stars, the magnitude difference being the same; (5) the theory of radiative equilibrium does not predict satisfactorily the masses of the companions. The first four peculiarities need no comments. But with regard to the masses of the companions the following remark must be made. Instead of the method employed above (§ 2), in the case of spectroscopic binaries we can compute the luminosity of the components, using the formula given by Eddington,

$$\text{Absol. magn. (bolom.)} = 4{\cdot}85 - 5 \log r/r_s - 10 \log T_e/T_s \qquad (4)$$

* Hagen, *Die Veränderlichen Sterne*, i., 729.

which is based exclusively on the photometric and spectroscopic data. On the other hand, the luminosity may be derived from Eddington's mass-luminosity relation. For stars of Table I (excluding H.D. 1337 as a star with exceptional mass) the average residual (observed absolute magnitude minus computed) is equal to $-0{\cdot}51$ for the primary and $-1{\cdot}17$ for the secondary. Excluding the last four stars as uncertain we reduce the average residuals to $-0{\cdot}34$ and $-0{\cdot}90$ respectively. Notwithstanding the uncertainties of the data we have more or less satisfactory agreement for the primaries. With regard to the secondaries the divergence is large (systematically negative residuals), and this cannot be wholly ascribed to the errors of observation. This result may be interpreted as evidence that the luminosity of the companions based on the photometric and spectroscopic data (r and J) is much larger than that based on the mass (from spectroscopic orbit) and mass-luminosity relation. This is in agreement with the negative residuals found for the mass-ratios (Table I). It seems probable that the companions are responsible for this peculiarity. But it is to be remembered that the relations found in eclipsing binaries cannot be seriously affected by these negative residuals.

8. With regard to the relation between the effective temperature of the components there is no direct contradiction between double stars and eclipsing binaries, since it was pointed out that the primaries belong to the main series and, therefore, the companion must be of a later type than the primary (Table V). But the observed advance of the spectrum of the companion for B, and partly for A, type cannot be considered as evidence against the hypothesis that the more massive stars evolve more rapidly. As concerns types F and G we have more serious evidence in favour of the double star by the scheme of evolution. However, the densities of the components of eclipsing binaries seem to be against it, since the density of the primary is usually larger; but the opposite cases are not rare (Table IX). It is evident that the conspicuous preponderance of systems in which the fainter companion is more tenuous is partly a matter of selection, and this suggests that there exists a great number of eclipsing binaries of small range in which the density of the fainter component is larger than that of the primary.

TABLE VIII.

Spectrum.	$\rho_1 > \rho_2$.	$\rho_1 \leqslant \rho_2$.
Bo–B9	17	6
Ao–A5	54	12
F	9	6
G	2	1
M	..	1

Though there are no data concerning the densities of the components of double stars, we are able to estimate very roughly the ratio of densities from the following formula which may be easily derived:

$$lg\rho_2/\rho_1 = log\, M_2/M_1 + 0{\cdot}6(m_2 - m_1) + 0{\cdot}6(J_1 - J_2)\,* \quad . \quad (5)$$

* The computed value for Sirius is equal, 102,000.

where J_1 and J_2 are the surface brightness of the components expressed in magnitudes. Using the data concerning double stars * and surface brightness derived by Seares, we have computed the ratio ρ_2/ρ_1 (Table IX).

Table IX.

Spectrum.	ρ_2/ρ_1.	(ρ_2/ρ_1).	n.
B	5·40	4·09	50
A	2·82	2·14	69
F	1·65	1·01	35
G	1·38	0·80	32
K	1·42	0·88	12
M	3·30	2·28	2

The results depend to a large extent on the assumed system of surface brightness. We give two values of the ratio of densities ρ_2/ρ_1 and $(\rho_2/\rho_1)'$ corresponding to the dwarf and giant system of surface brightness. It is evident that with decreasing surface brightness ρ_2/ρ_1 decreases, the other circumstances being the same. We have reason to think that the surface brightness of both components in double stars is more or less normal.† It seems to be probable that with regard to the companions there is a radical difference between double stars and eclipsing binaries, since the difference in spectra or surface brightness of the components in the latter is much larger than that in double stars, the magnitude difference being the same. Probably there is a radical difference in the ratio of the densities of the components in these two classes of binaries that may be partly connected with the former peculiarity. This emphasizes the importance of observations of colour-indices and spectra of the components of double stars and eclipsing binaries. It is not difficult to understand that the low density of the companions and their small surface brightness is closely connected with the ratio of radii (in most cases $r_2/r_1>1$), which is directly derived from the light-curve.

It was shown above that eclipsing binaries belong to the main series, and for them we have found many analogies with double stars. But it is difficult to understand why there are known among the eclipsing binaries no systems with the primary of a later type than the secondary, which are very frequent among double stars. It appears that this difference cannot be accounted for by selection, and the cause of this is probably of an intrinsic character. This emphasizes the importance of securing observations of late type eclipsing binaries. By the way the giant eclipsing binaries of late type with known orbit (ϵ Aurigæ, W Crucis, RZ Ophiuchi, Rχ Cassiop.) are associated with long period, unusual for the eclipsing binaries.

If there is no difference between eclipsing and spectroscopic binaries of the main series (majority of B and A type) we must find the

* *M.N.*, **85**, 250.

† *Public. Pulkovo Observ.*, 1925, x., No. 95 and *Abetti. Public. Observ. Arcetri*, **40**, 7.

spectrum of the secondary to be of type F and G. In the light of the data of Table V the possibility of observing the spectrum of the secondary in spectroscopic binaries is very limited. It is hardly possible to trace the spectrum of the secondary, if the difference in visual brightness of the components is more than $0{\cdot}5^{m}$. It is not surprising, therefore, that the percentage of systems with both spectra among the spectroscopic binaries is very small. The observational data are scanty, but it is established with more or less accuracy that when two spectra are substantially equal in brightness, they are identical in class, and when one spectrum is considerably fainter, the spectrum of the secondary is apparently of a slightly earlier class than that of the primary; more often no remark is made. It is obvious that allowance must be made for the selection which is favourable for such a relation. In eclipsing binaries, on the contrary, the discovery of systems with the secondary of a later type is more probable, the other circumstances being the same, since the primary minimum will be photographically deeper. This difference in the selection in both classes of stars must be recognized. The great percentage of spectroscopic binaries with the components of identical or nearly identical spectra may be partly accounted for by the reflection effect.* But the spectroscopic binaries with the secondary of an earlier type than the primary are of great importance, since such systems among the eclipsing binaries are known only as exceptions. For this reason it is highly desirable to revise all observations concerning the spectra of the components of spectroscopic binaries.

It is remarkable that the well-known luminosity spectrum diagram of Russell-Hertzsprung may be derived exclusively from data concerning the spectrum magnitude relation of the components of double stars. Moreover, this diagram may be interpreted as evidence that the less massive component is further advanced in the scheme of evolution. Such a trial to represent the components of the eclipsing binaries leads to another diagram which has but little in common with the Russell-Hertzsprung scheme. The mutual relations between spectra, densities, and masses of the components in both classes of binaries, which appears to indicate undoubted evolutionary progress within each of the classes, cannot be represented by a common scheme of evolution. It is evident that this scheme is much more complicated than it has usually been supposed.

Summary.

The mass-ratio computed on the basis of the theory of radiative equilibrium for 15 eclipsing variables known to be spectroscopic binaries is found to be in fair agreement with its observed value; it decreases with increasing difference in brightness of the components (Tables I and I′).

The discussion of 108 eclipsing binaries leads to the conclusion that the mass-ratio is closely connected with the spectrum of the primary, and tends to approach unity with advancing type (Table II).

The greater the magnitude difference of the components, the more

* Eddington, *I.C.S.*, p. 214.

the components are different with respect to surface brightness or spectral class (Tables III and IV).

The effective temperatures of the secondaries deduced from the ratio of surface brightness decrease with advancing type of the primary (Table V).

The density of the components increases with advancing type, and the ratio ρ_2/ρ_1 tends to unity, except for the B-type stars (Table VI).

The primaries are shown to belong to the main series of stars ; with respect to the secondaries the data are contradictory.

The computed effective temperatures and the observations show that the secondaries are of a much later type than the primaries.

The Russell-Hertzsprung diagram, if constructed on the basis of the luminosity-spectrum data for eclipsing binaries, would be of a very unusual aspect.

The correlations mentioned above show an analogy with those found in double stars.

Postscript.—The foregoing paper was ready for press when I received the *Astronomical Journal*, No. 889, November 1927, with the paper by B. McLaughlin on the masses and luminosities in eclipsing binaries. Except that some conclusions are the same, the papers have but little in common.

Simeis, Branch Observatory of Pulkovo :
1927 *December* 15.

Comet Skjellerup (1927 *k*). By P. R. Chidambara Aiyar. (Plate 3.)

[The following is an extract from a letter to the Astronomer Royal from P. R. Chidambara Aiyar, in confirmation of a cablegram sent 1927 December 15.]

I believe that I shall be expected to state something about how this remarkable daylight comet came to be noticed by me and what observations I have been able to subject it to.

As Dr. Royds, the Director of the Observatory, was not in the station, I was the assistant engaged in taking spectroheliograms of the Sun on the morning of the 15th inst. At 8.30 a.m. (I.S.T.), my attention happened to be directed to a bright object near the Sun. As there were small patches of cloud floating in the neighbourhood of the Sun, I first took this also to be one such, but in a moment it became quite plain to me that I was looking at a brilliant comet near the Sun. It was, as estimated by the eye, about $2\frac{1}{2}$ apparent diameters of the Sun distant from his nearest edge to the east and a little to the south. I at once made a sketch of it, and also wired to the Madras Observatory to observe and verify the apparition. Strange to say, it could not be observed in a telescope that day, but as there was some change in its appearance, I made another sketch of it at 12.10 p.m. From 2.30 p.m. clouds made further observations impossible. I am enclosing herewith a copy

N

E

W

Bright Comet near Sun $8^h\ 35^m$ a.m., I.S.T.
1927 Dec. 15.

Bright Comet near Sun $12^h\ 10^m$ p.m., I.S.T.
1927 Dec. 15.

Sketched by P. R. Chidambara Aiyar, Assistant, Kodaikanal Observatory.

of my sketch of its naked-eye appearance on the 15th inst. The head of the comet was much brighter and larger than Venus seen some distance ahead in the west.

On the morning of the 16th I again made a sketch of it as it was visible to the unaided eye. It had got separated from the Sun, had become northerly from it, and had become less bright. I made a series of sketches as it was seen in the smaller telescopes of the Observatory, which revealed it with a remarkable wealth of detail. It was a magnificent object to look at. The head was a nebulous mass with an extremely bright nucleus from which emanated two luminous, curved arms. They were rather unsymmetrical, one being more curved than the other. The tail was bifurcated and enormous, not so much in length as in bulk. This was before noon. In the afternoon the tail increased in length as well as in volume to such an extent that I thought it was a huge giantess who had let loose the terrific glory of her copper-coloured mass of tresses, and who was running away with her back turned against us. By this time symmetry had begun to be established between the two portions of the tail as well as in the internal arrangements in the head.

I was able to note its positions on the 16th, 17th, and 18th, which are given below. On the 19th the sky was thick and no observations were possible. I hope that the Madras Observatory has been able to keep it under observation, and I have requested that the records of observations be sent to me. My cablegram should have reached you on the 16th in the usual course, and I trust that observations have been possible since then, especially as it has moved north of the Sun.

The comet was plainly visible to the naked eye on the 15th, 16th, and 17th, and was quite invisible on the 18th. I hope that by now several observatories have picked it up and have been observing it.

Position of Comet.

Date.	I.S.T.	R.A.	Dec.
16.12.27	$12^h\ 0^m$	$17^h\ 37^m$	S. 20° 7′
17.12.27	$10^h\ 10^m$	$17^h\ 43^m$	S. 17° 20′
18.12.27	$11^h\ 14^m$	$17^h\ 46^m$	S. 14° 43′

Observation of Comet Skjellerup (1927 *k*). By J. Marks, 4th Officer, s.s. *Cumberland*.

(Communicated by the Secretaries.)

On December 6 at $7^h\ 36^m\ 51^s$ G.M.T., in Lat. 36° : 12′ S., Long. 172° : 44′ W., observed a comet bearing S. 30° : 38′ W. (T). Altitude of nucleus 6° : 45′. Length of tail 1° : 10′. Altitude of end of tail 7° : 18′. Angle between nucleus and α Centauri 19° : 23′. Altitude of α Centauri 7° : 51′.

The weather was perfectly clear at time of observations, but vessel was rolling easily to a moderately heavy southerly swell. Clouds were Alt. St, amount one. Altitudes were uncorrected. Height of eye 42′, no index error to sextant. Dip 6′ 16″.

The comet, of which we have no record on board, consisted of a moderately bright nucleus of about the third magnitude, and had a tail resembling Bredikine's Second Type, very clearly defined for the first degree of its length, after which it appeared to become very indefinite although the curvature was quite apparent. It clearly extended ahead of the nucleus and completely surrounded it. The tail, if extended, would have passed very near to α Argus.

On the same day at $14^h\ 9^m$ G.M.T., it was again observed rising, and bore S. 29° E. (T.), Alt. 4° : 47′. At $15^h\ 7^m$ G.M.T., it bore S. 35° E. (T.), Alt. 10° : 12′. Daybreak prevented further observations, and for the remainder of the voyage the sky was overcast.

[Dr. Crommelin has computed that at the time and position given, the true altitude of α Centauri was 8° 33′·5. The observed altitude 7° 51′ when corrected for refraction − 6′ 36″ and dip − 6′ 16″ gives only 7° 38′, which differs nearly 1° from the computed. SECRETARIES.]

Occultations of Stars by the Moon, observed at the Nizamiah Observatory, Hyderabad, during the period 1926 *January–*1927 *September.*

(*Communicated by the Director.*)

No.	Date.	Star.	Mag.	Phase.	Greenwich Mean Time. h m s	Observer.
1	1926 Jan. 23	δ Tauri	3·9	D	17 47 47·8	B
2	23	δ Tauri	3·9	R	19 9 3·0	B
3	23	64 Tauri	4·9	D	18 36 20·6	B
4	23	64 Tauri	4·9	R	19 41 48·9	B
5	28	ε Cancri	6·3	D	16 15 38·6	B
6	June 16	A.G.C. Leipzig 4049	7·9	D	14 14 15·2	B
7	Oct. 24	ε Tauri	3·6	D	1 48 57·2	M
8	Nov. 13	A.G.C. Wash. 8314	8·6	D	15 5 36·5	B
9	15	30 Piscium	4·7	D	12 24 45·2	B
10	15	30 Piscium	4·7	R	13 36 45·2	B
11	Dec. 26	C Virginis	5·1	D	22 55 10·4	M
12	27	C Virginis	5·1	R	0 24 44·0	M
13	1927 Jan. 8	A.G. Wien 8348	9·1	D	13 57 20·8	B
14	8	B.D. − 19°·6215	9·4	D	14 6 34·3	B
15	8	A.G. Wien 8357	9·1	D	14 52 38·3	B
16	9	A.G. Strassburg 98	9·0	D	14 23 38·3	B
17	9	B.D. − 3°·55	9·3	D	14 24 51·8	B

No.	Date.	Star.	Mag.	Phase.	Greenwich Mean Time. h m s	Observer.
18	1927 Jan. 9	A.G. Strassburg 101	7·1	D	14 42 43·3	B
19	10	A.G. Albany 365	8·7	D	13 5 49·5	B
20	10	A.G. Albany 366	9·1	D	13 44 4·0	B
21	12	147 B Arietis	5·8	D	13 27 42·2	M
22	12	147 B Arietis	..	R	14 57 49·5	M
23	13	162 B Tauri	6·3	R	14 46 23·3	M
24	Mar. 6	ν Piscium	4·7	D	15 22 12·3	M
25	7	Anon.	..	D	14 25 13·0	M
26	8	Bord. ph. 16° 3^h 20^m 109	9·0	D	13 33 18·6	B
27	8	Bord. ph. 16° 3^h 20^m 105	10·0	D	14 40 7·0	B
28	8	Bord. ph. 16° 3^h 20^m 110	10·2	D	14 47 56·9	B
29	9	A.G. Berlin II. 1167	9·0	D	14 32 23·1	B
30	9	Anon.	..	D	15 37 44·3	B
31	9	A.G. Berlin II. 1187	8·4	D	16 1 8·0	B
32	9	A.G. Berlin II. 1198	7·7	D	16 39 3·5	B
33	11	A.G. Berlin I. 2353	8·7	D	16 49 1·2	B
34	11	36 B Geminorum	6·0	D	17 24 38·2	B
35	11	A.G. Berlin I. 2366	7·8	D	17 25 11·7	B
36	May 7	A.G. Berlin I. 3410	8·5	D	14 39 45·2	B
37	7	A.G. Berlin I. 3420	8·9	D	15 12 41·2	B
38	June 5	A.G. Berlin II. 3987	9·0	D	14 47 33·1	B
39	6	A.G. Leipzig 4104	6·5	D	15 36 34·1	B
40	14	ν Scorpii B	7·0	D	19 22 11·7	M
41	14	ν Scorpii A	4·1	D	19 22 53·7	M
42	14	ν Scorpii A	4·1	R	20 47 39·6	M
43	Sept. 6	Cordoba A 12970	8·4	D	15 1 16·8	B

Instrument used :

15-inch Grubb refractor throughout, except for No. 24, which was observed with a 3-inch telescope.

Observers :

B = T. P. Bhaskaran.
M = M. K. Bappu.

Notes.

1, 3, 7, 21, 41 : instantaneous.
10 : probably slightly late.
2, 4, 5, 10, 22, 34 : fair.
42 : time somewhat uncertain.

Nizamiah Observatory,
Hyderabad, India.

Occultations of Stars by the Moon, observed at the Astronomical Observatory, Kiew, in the year 1927; *and Micrometer Observations of the Position of Jupiter.* By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

No.	Date 1927.		G.M.T. h	m	s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.	
1	Jan.	13	16	53	41·36	180 B Tauri	6·1	D	R 145	T	
2	Feb.	9	20	52	28·37	162 B Tauri	6·3	D	R 145	T	
3		9	20	52	29·04	162 B Tauri	6·3	D	B 61	L	
4	Apr.	7	17	2	1·04	73 Geminorum	5·6	D	R 145	T	
5		7	18	7	13·24	73 Geminorum	5·6	R	R 145	T	
6		7	18	21	55·92	76 Geminorum	6·3	D	R 145	T	
7		7	18	21	56·24	76 Geminorum	6·3	D	K 24	D	
8	June	5	18	47	8·70	107 B Leonis	6·3	D	R 145	T	
9		5	18	47	9·23	107 B Leonis	6·3	D	K 24	D	
10		5	18	47	9·49	107 B Leonis	6·3	D	B 61	L	
11	Sept.	3	18	33	42·18	β_1 Scorpii	2·9	R	R 145	T	
12		9	20	28	18·81	154 B Capricornis	6·1	D	R 145	T	
13		10	19	16	50·46	69 Aquarii	5·6	D	R 145	T	
14		10	20	24	1·42	τ Aquarii	4·2	D	R 145	T	
15		10	20	24	2·25	τ Aquarii	4·2	D	B 61	L	
16	Oct.	3	17	31	50·09	λ Sagittarii	2·9	D	R 145	T	
17		3	17	31	50·65	λ Sagittarii	2·9	D	K 24	D	
18		3	17	31	50·57	λ Sagittarii	2·9	D	B 61	L	
19		14	19	2	13·78	ι Tauri	4·7	D	R 145	T	
20		14	19	2	12·32	ι Tauri	4·7	D	B 61	L	
21		14	19	55	14·18	ι Tauri	4·7	R	R 145	T	
22		14	19	55	14·20	ι Tauri	4·7	R	B 61	L	
23	Dec.	7	16	18	3·11	163 B Tauri	5·8	D	R 145	T	
24		7	16	18	3·67	163 B Tauri	5·8	D	B 88	L	
25		8	17	0	22·59	B.D. + 21°754	8·4	D	R 145	T	During the Total Lunar Eclipse.
26		8	17	11	7·63	ι Tauri	4·7	R	R 145	T	
27		8	17	35	39·60	B.D.+21°756	9·5	D	R 145	T	
28		8	18	2	21·21	B.D.+21°760	9·5	D	R 145	T	
29		8	18	8	48·15	B.D.+21°761	9·5	D	R 145	T	

Instruments used:

R = 243-mm. Merz-Repsold refractor.
K = 120-mm. Utzschneider and Fraunhofer comet-seeker.
B = 81-mm. Busch refractor.

Observers:

T = S. Tscherny.
D = M. Ditschenko.
L = E. Laurentjewa.

Notes.

Nos. 1, 12, 14, 16, 26: instantaneous.
Nos. 2, 4, 5, 6, 8, 11, 13, 15, 18, 21, 23, 25, 27: good.
Nos. 19, 24, 28: satisfactory.
Nos. 20, 29: doubtful.

On 1927 December 7 the position of the centre of Jupiter relative to the star B.D. − 3°5697 was observed by Professor Tscherny with the filar-micrometer.

The observations were reduced as in *M.N.*, **84**, 772, and gave the apparent geocentric position of Jupiter for $16^h\ 46^m\ 1^s{\cdot}63$ G.M.T. as

$$23^h\ 40^m\ 44^s{\cdot}58, -3^\circ\ 35'\ 2''{\cdot}2,$$

the position of the star being assumed to be (*B.A.A.H.* 1927)

$$23^h\ 40^m\ 49^s{\cdot}67, -3^\circ\ 34'\ 36''{\cdot}2.$$

This gives corrections to Newcomb's tables (*A.E.*) as $-0^s{\cdot}18$, $+2''{\cdot}6$, and to Le Verrier's tables (*C. d. T.*) as $-0^s{\cdot}19$, $+2''{\cdot}6$.

Astronomical Observatory, Kiew:
1928 *January* 6.

Observations of the Transit of Mercury on 1927 *November* 10, *at Hong Kong Observatory.*

(*Communicated by the Director.*)

Ingress—

	h	m	s	
Internal contact . . .	11	3	52	Observer, T. F. C.
,, ,, . . .	11	3	38	,, B. D. E.
Mean	11	3	45	
Computed time . . .	11	4	29	
Correction to computed time .			− 44	

Egress—

	h	m	s	
Internal contact . . .	16	26	30	Observer, T. F. C.
,, ,, . . .	16	26	7	,, C. W. J.
,, ,, . . .	16	26	50	,, B. D. E.
Mean	16	26	29	
Computed time . . .	16	27	26	
Correction to computed time .			− 57	
External contact . . .	16	28	7	Observer, T. F. C.
,, ,, . . .	16	27	58	,, C. W. J.
,, ,, . . .	16	28	30	,, B. D. E.
Mean	16	28	12	
Computed time . . .	16	29	7	
Correction to computed time .			− 55	

The initials, T. F. C., C. W. J., and B. D. E., are those of Messrs. Claxton, Jeffries, and Evans respectively.

The Transit of Mercury, 1927 November 10, observed at Stonyhurst College Observatory. By Rev. E. D. O'Connor, S.J.

The transit of Mercury was observed under moderate conditions on November 10, the image of the Sun being projected on to the Sun-spot board. Definition was bad and the image "boiling" violently. There was a certain amount of cloud present, but the Sun was clear at time of third and fourth contacts. Using the data given in the *Nautical Almanac*, p. 459, the result worked out at

$$O - C = -18^s \text{ for third contact,}$$
$$O - C = -29^s \text{ for fourth contact,}$$

O being the observed time, and C the calculated time.

Observations of Mercury Transit on 1927 November 10. By Issei Yamamoto, D.Sc., Professor of Astronomy, Kyoto University.

Early in last November I made a trip to Taiwan (Formosa) Island to observe the complete phase of the Mercury Transit on the solar disc on November 10. On arriving at Keelung port, I decided at the suggestion of the authorities of Taihoku Meteorological Observatory to erect my instrument (10 cm. Heyde equatorial) at the Meteorological Station of Taityu city. The geographical position of the Taityu Station is

Long. = 120° 41′, E. of Greenwich. Lat. = 24° 9′, N.
Height above sea-level = 77 metres.

The weather was very favourable on the day of the Transit; the sky was perfectly clear except a few hours in the early morning, when it was overcast with light cirrus. The wind was calm to a light breeze from N.N.W. After some trials, I made the following observations of contact times by visually observing the solar disc with full aperture of 10 cm. and a power of 133.

	h	m	s	s	
1st contact .	11	2	9	± 7	(suspected),
	11	2	16	± 5	(surely seen).
2nd contact .	11	3	48	± 2	(geometrical),
	11	3	51	± 2	(optical; disapp. of black drop).
3rd contact .	16	26	54	± 2	(optical; black drop connection),
	16	27	0	± 3	(geometrical).
4th contact .	16	28	31	± 5	(suspected),
	16	28	38	± 5	(surely gone).

The time is in Japanese Western Standard Time (8 hours in advance of Greenwich Mean Time). The chronometer correction was obtained

by comparing with that of the Taihoku Meteorological Observatory where astronomical observations are made.

The solar image was quite steady, with no perceptible boiling of the limb at the time of the first two contacts. When the last two contacts were observed, the sun was fairly low in the west and a slight sign of boiling was seen on the limb.

I assumed a relative weight $\frac{1}{10}$ to each of observations of the 1st and 4th contacts, and obtained the following result as the most probable value of the difference between computation and observation of the contacts :

$$-32^{s}\cdot2 \text{ (observation earlier than computation).}$$

In the interval between the 2nd and 3rd contacts, I tried to photograph Mercury on the Sun with the same instrument, and three plates were successful.

Kyoto University Observatory,
1927 December 23.